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Essays on electricity transmission investment and financial transmission rights

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ABSTRACT

The U.S. electric power industry has been going through fundamental restructuring and realignment since the 1990's. Many issues and problems have emerged during the transition, and both economists and engineers have been looking for the solutions fervently. In this dissertation, which consists primarily of three essays, we apply economics theory and techniques to the power industry and address two related issues, transmission investment and financial transmission rights (FTRs). The first essay takes the decentralized perspective and investigates the efficiency attribute of market-based transmission investment under perfect competition. We clarify, for the first time, the nature of the externality created by loop flows that causes transmission investment to be inefficient. Our findings have important implications for better understanding of transmission market design and creating incentives for efficient transmission investment. In the second essay, we define several rules for allocating transmission investment cost within the framework of cooperative game theory. These rules provide fair, stable or efficient cost allocations in theory and are good benchmarks against which the allocation mechanism in practice can be compared and improved upon. In the last essay, we make exploratory efforts in analyzing and assessing empirically the performance of the Midwest independent system operator (MISO) FTR auction market. We reveal some stylized facts about this young market and find that it is not efficient under the risk-neutrality assumption. We also point out and correct the drawbacks in previous related work and suggest about more complete empirical work in future. In all, this dissertation makes both theoretic and empirical analysis of the two hot issues related to the power industry and comes up with findings that have important implications for the development of this industry.

CHAPTER 1. OVERVIEW

The contribution of electric power to modern life is unparalleled in the U.S. and almost all the other countries in the world. Electricity is pervasive in our society, permeating every aspect of the economy and affecting our daily lives both in business and at home. We depend on extraordinarily high reliability in electricity services. Interruptions are limited to no more than a few hours per year and have far-reaching consequences when they occur.

This dependence on electricity is starkly evident as we recall the blackout in the eastern part of North America in the summer of 2003. The costs are estimated in billions of dollars and still have not been fully figured out. This event underscores the extreme importance of electricity reliability and the unsubstitutable role that electricity plays in the economy and people's lives. As a response to the electricity failure, public commissions, regulatory bodies, and federal and state agencies have undertaken fervent activities to identify appropriate actions to prevent such outages in the future.

Not only are we dependent on electricity, but customers' expectations of reliability have changed. Consumers are demanding very high quality power in their homes for electronics and at work for industrial processes. Some industries have indicated that the tolerances of their processes require extremely high reliability in electricity supply. Therefore, it is of great importance to ensure reliable electricity and transmission services. This is highlighted in the context of an unprecedented deregulation of the electric power industry.

During the past two decades, the way in which electricity is provided to customers has changed fundamentally in the U.S. The changes do not speak to the physics of electricity or to how it is delivered in a physical sense, but they affect the institutions, pricing, reliability and regulation of this essential service. Previously, the power industry was one of the most heavily regulated in the U.S. It was characteristic of a structure dominated by vertically integrated utilities, regulated primarily at the state level. That is, the functions of generation, transmission, and distribution were responsibilities of a single entity. Restructuring and realignment, which started in the early 1990's has altered the rules that governed control, operation, ownership, and regulation of the industry. The traditional integrated utility has been disaggregated. Generation is now controlled or owned and operated by private, nonregulated companies. Electricity prices, instead of being set by regulators, are determined by supply and demand in the market. States have moved away from regulations that set rates for electricity and toward oversight of an increasingly deregulated industry. Transmission, however, remains regulated as a natural monopoly to ensure open and non-discriminatory access, a central component of the competitive electricity market.

The wholesale electricity market design that is being practiced in different regions in the U.S. is the Wholesale Power Market Platform (WPMP) rooted in Hogan (1992). California, the Mid-Atlantic States, New England, New York, the Midwest, and the Southwest have adopted or implemented this design to some extent. The central idea of the proposal is the operation of wholesale power markets by Independent System Operators (ISOs) or Regional Transmission Organizations (RTOs) using locational marginal pricing to price power energy. To be specific, load serving entities (LSEs) and generators submit to the ISO bids and offers, respectively, which reflect their supply and demand curves. Then the ISO determines the power dispatch and the locational marginal price (LMP) at each node for each hour, by solving the security-constrained economic dispatch problem (SCED). Basically, the ISO maximizes the total social surplus subject to a set of constraints.

In this dissertation, we focus on the U.S. electric power industry and address two related issues under hot debate in the transition process: Transmission investment, and financial transmission rights (FTRs). People have been looking for or creating incentives for encouraging efficient investment in transmission expansion. One of the proposals is that FTRs might be able to do this job.

(1) Transmission investment

Among the many challenges that need to be overcome in the restructuring process, those revolving around transmission expansion and investment have turned out to be the most debatable and intractable. Transmission expansion planning is the process of deciding how and when to invest in additional transmission facilities. These decisions have significant consequences on the reliability and efficiency of the future power system. In addition, they usually involve large capital expenditures and complex regulatory processes. Previous to deregulation, the necessary coordination between the two highly independent functions, namely generation planning and transmission planning was carried out in an intentionally integrated fashion, often involving the same people, targeting the objectives of the organization's management to whom the analysts and decision-makers reported. Transmission enhancements that affected multiple utilities were handled through bilateral coordination or through well-structured coordinating bodies. The utility paid for transmission upgrades and recovered regulatorily approved costs through customer rates. Under deregulation, the number of organizations involved in generation planning and transmission planning is significantly increased, each with their own objectives. Generation is planned by a multiplicity of companies seeking to maximize their individual profits through energy sales, while transmission is planned by transmission owners seeking to maximize their profits through transmission services, all overseen and coordinated by a centralized authority, usually ISO or RTO, seeking to ensure grid reliability and market efficiency. The increased number of stake holders requires procedures for coordinating among them the necessary analyses, decisions, and financial implications. Besides, it motivates the need for incentives so that organizations perceive transmission investment and ownership to be attractive.

There has been insufficient investment in transmission expansion relative to growing demands and generation capacities. Different approaches to attract or incentivize efficient or optimal transmission investment have been put forward. They all fall somewhere between the two extremes: completely regulated transmission investment and purely market-based transmission investment. The consensus has been reached that we can not solely depend on markets for optimal transmission investment, and some sort of regulation and supervision is necessary, although the extent of centralization versus decentralization in transmission investment is still an open question.

To create incentives for efficient transmission investment, we need to develop an in-depth understanding of transmission market design, to which this dissertation makes its contribution. The literature tends to emphasize that the reasons for necessity of regulation mainly lie in market power in wholesale electricity market and natural monopoly in transmission market. Admittedly, the existence of these factors does cause market failure in power and transmission markets. However, this is true for any other market as well, so does not say anything specific to the power industry. Different from most of the existing paper, Chapter 2 of this dissertation clarifies the nature of an externality unique to the power industry, which has never been clearly identified before. Due to this externality, market-based transmission investment may be inefficient. Our finding facilitates understanding of transmission market design and has important implication for defining property rights to induce efficient transmission investment.

The externality we find and other reasons mentioned earlier entail regulation and overseeing by a central entity on transmission investment. Market itself will not do the right job. One of the responsibilities of such an entity should be to allocate transmission investment costs when private incentives fail to cover the expenses of socially beneficial investment projects. In Chapter 3, we address the situation in which transmission investment enhances the social welfare, but does not benefit market participants equally, such that a central authority is needed to decide and impose a proper cost allocation among them in order to have the investment undertaken. We use the insights of cooperative game theory to define several allocation rules. Each rule provides reasonable cost allocations to the electricity cost allocation problem in the situation described above and makes a benchmark against which the industry practices can be compared.

Chapters 2 and 3 both address transmission investment, but they take different perspectives. Chapter 2, from the decentralized point of view, studies whether transmission investment induced by market is optimal. In contrast, Chapter 3 takes a regulatory perspective and inquires what needs to be done by a central planner. These two Chapters together make a complete analysis concerning the combination of market and regulator in transmission expansion investment.

(2) FTRs

Transmission rights stand at the center of market design in the restructured power industry. The industry searched for many years without success looking for a workable system of physical rights that would support decentralized decisions controlling use of the grid. In the design built on the centerpiece of a coordinated spot market, physical transmission rights or any associated scheduling priority would create perverse incentives and conflicts with priority defined by the bids used in a security-constrained dispatch. Since physical rights will not work, something different is needed to achieve the same objective in providing a compatible definition of transmission rights for a competitive electricity market.

Congestion that causes electricity prices to differ across nodes, leads to the interest in financial transmission rights (FTRs) used as a hedging tool against congestion and price uncertainty. A coordinated wholesale market with LMPs complemented by FTRs is a hallmark of market design that works. Now it is a common practice in the U.S. wholesale power market for ISO to issue FTRs. An FTR is a financial instrument that entitles the holder to compensation for transmission congestion costs that arise when the transmission grid is congested. The amount of compensation is based on the differences in the day-ahead LMPs. They do not protect market participants from congestion charges related to scheduling power in the real-time market or deviating from the day-ahead schedule. Nor do they hedge against transmission loss charges. Besides, FTRs are independent of the physical power dispatch. The FTR holder has the financial right to the congestion rent between two specified nodes regardless of the actual energy deliveries.

According to the literature, there are four types of FTRs: point-to-point (PTP) obligation, PTP option, flowgate (FG) obligation, and FG option (Hogan 2002). An FTR obligation entitles its holder to a positive revenue when the day-ahead congestion occurs in the direction as defined by the FTR and to a negative revenue (i.e. to a loss) when the day-ahead congestion occurs in the opposite direction. An FTR option, in contrast, never results in negative revenue, because when the congestion happens in the opposite direction, the FTR option holder is not obligated to pay. Therefore, other things being equal, the FTR option has a higher value than that of the obligation.

Up to now, FTRs have been widely used in major U.S. wholesale electricity markets, such as the Pennsylvania, New Jersey and Maryland (PJM), New York and California markets. Some empirical work has been done on those relatively mature markets to evaluate the efficiency of FTR market and test the validity of the underlying theory. The main finding is the inefficiency in FTR market practice. More recently, in April 2005 the Midwest ISO (MISO) kicked off its wholesale electricity market together with the FTR market. Little effort, however, has been made to analyze the performance of this new FTR market. Data availability is one of the reasons for scarcity of such studies. In Chapter 4, we take the initiative in analyzing this transient market theoretically and empirically. Using the data of LMPs and MISO monthly FTR auction results in the one-year period April 2005-March 2006, we assess the performance of the MISO FTR market so far. At the same time, we point out the flaws in previous empirical studies on FTR markets. The limited data make it impossible to make complete analysis or reach definite conclusions and we have to make some simplifying assumption in our studies. But to address this problem, we indicate what data will be needed for further studies.

Chapter 4 is related to the preceding two chapters, as FTRs, apart from being a hedging instrument, have also been argued to be potential, effective incentives for transmission expansion. Whether or not FTR markets work well in practice affects the possibility of using FTRs for transmission investment incentives.

Here is the outline of the dissertation. In the current chapter, we provide some background information of our research. Chapter 2 investigates the efficiency attributes of market-based transmission investment and clarifies the nature of the externality created by loop flows that can cause inefficient transmission investment. We study the allocation of transmission investment cost and propose several cost allocation rules within the framework of cooperative game theory in Chapter 3. In Chapter 4, we analyze FTRs and the MISO FTR market and evaluate its performance and efficiency. The main conclusions from Chapters 2-4 are summarized in the final chapter.

CHAPTER 2. MARKET-BASED TRANSMISSION INVESTMENT UNDER PERFECT COMPETITION: IS IT EFFICIENT?

2.1 Introduction

Since the 1990's, the U.S. power industry has been going through a fundamental restructuring from heavy regulation to competition. Transmission networks play a critical role in providing access to all participants in a competitive market for supply and delivery of electric power. A more robust transmission system would bring in competitive bidders from far away and eliminate the chance of dominant generators exercising market power due to transmission constraints. Reality is that investment in transmission expansions has been insufficient relative to the needs for expanding generation capacity and growing demand. Lack of transmission investment limits our ability to maintain or improve electric reliability, accommodate growing loads and incorporate higher generation capacities. It is necessary and urgent to develop a transmission network that enhances efficiency of a competitive market. To do that, we need to find solutions with regard to signals and incentives for encouraging efficient investments in transmission expansion.

Power generation and electricity marketing are generally considered to be areas in which competition might work and deregulation has taken place. Transmission, in contrast, is still natural monopoly and a limited amount of merchant transmission investment has been forthcoming to date in electricity markets where it is permitted and encouraged (Joskow and Tirole (2004)). There has been an intense debate regarding the best way to attract or incentivize investment in transmission and different approaches have been proposed. Some take a more decentralized manner and argue for merchant or market-based transmission investment. The other, from a more centralized point of view, emphasizes the importance of regulation in transmission investment. Hogan (1992) proposes a contract network pricing model, using congestion payments as the rental fee for use of the capacity rights. Within this contract network regime, Bushnell and Stoft analyze the potential of "transmission congestion contracts (TCCs)" being an incentive for grid investment. In Bushnell and Stoft

(1996a), they show that under certain conditions the contract network approach can effectively deter detrimental investments. They formalize a rule for allocating TCCs to those who provide grid improvements that might allow a decentralized, profit-driven market to carry out efficiently the difficult function of grid modifications. Following these, Bushnell and Stoft (1997) outline a process by which transmission planning and investment would be undertaken by competitive entities in a lightly regulated environment and analyze how the network externalities can be managed successfully by the system proposed. The analyses in those papers arguing for market-based transmission investment are mainly based on assumptions equivalent to the ones of a model of perfect competition. In a recent paper, Joskow and Tirole (2005) points out that those assumptions exclude several attributes of power markets and transmission networks, such as market power in wholesale electricity markets, lumpiness in transmission investment opportunities and stochastic attributes of transmission networks, etc. The authors conclude that without the perfect competition assumptions, inefficiencies may result from reliance on the merchant transmission investment framework. Gans and King (1999) find that current options of market-driven investment are unlikely to be adequate in terms of encouraging socially optimal levels and timing of new transmission investment. As an alternative, they propose a regulatory scheme to overcome that problem. Shang and Volij (2004) address cost allocation of transmission investment from the perspective of cooperative game theory. They identify the situation where transmission investment will benefit society as a whole, but not every market participant. In this case, no coalition is prepared to undertake the investment and a regulatory decision to approve the investment and allocate the costs is required. Leautier (2000), Grande and Wangesteen (2000) and Vogelsang (2001) focus on the design of economic regulatory mechanism for Transcos. The main idea is that an incentive-compatible regulatory mechanism for a Transco must provide incentives to the regulated firm to make efficient investment decisions, and must also permit it to earn enough revenues to cover its cost. Although the extent of regulation versus deregulation in transmission investments is still under hot debate, it is now agreed that we can not solely count on market for such investments and that regulation is needed to achieve efficient investment. Hogan (1999) emphasizes that with TCCs to allocate transmission benefits, it would be possible to rely more on market forces, partly if not completely, to derive transmission expansion. Hogan (2002b) generalizes Bushnell and Stoft's analysis and makes a preliminary attempt to analytically provide some axioms to properly define LT FTRs (long-term financial transmission rights)1. He declares that reliance on merchant investments may not cover all cases, but it could provide an efficiency improving complement to regulated, rate-based transmission investment. The main approaches of attracting investment in the long-run transmission expansion are discussed in Rosellon (2003).

So far, the most often cited reason for the necessity of regulation in transmission investment is that market power in the electricity and transmission markets will cause market failure. As mentioned just now, Joskow and Tirole (2005) claim that due to the attributes such as market power and economies of scale, market-based transmission investment may result in inefficiency. However, the existence of those attributes can lead to market failure in any market. They are neither special to nor inherent in the power market. If they were the only cause for transmission investment inefficiency, there would be no need to single out the power industry for intensive research, and we could simply borrow the recipe from the other industries. What is of greater significance, actually, is something inherent in or unique to power transmission, if there is any. Knowing them help us understand power transmission market design principles better and find solutions to incentivizing efficient transmission investment. In this paper, we identify and address one such thing that should be given more notice.

It is well known that generation of power can be efficiently decentralized by means of a price system and competitive markets. Indeed, Chao and Peck (1996) show that for a fixed grid, a competitive equilibrium is efficient. In other words, the equilibrium nodal and transmission prices induce an efficient dispatch. It is also known that this result breaks down as soon as the grid itself is endogenous (Bushnell and Stoft (1996a, 1997)). That is, there is a market failure in the power market once investment in transmission is allowed. The alleged reason for this market failure is the externalities created by loop flows2. That loop flows are responsible for the market failure in transmission investment is clear, since a power market

[1] FTRs and TCCs are the same thing under different names
[2] There are externalities when the actions of one agent **directly** affect the payoff of another agent associated with a fixed action.

with endogenous investment in a radial network can be efficiently decentralized through a market mechanism3. However, the nature of the externalities created by loop flows has, to the best of our knowledge, never been identified. In most papers, these externalities are either taken for granted or addressed ambiguously. Only Chao and Peck (1996) touches the brink by mentioning that "new investments in transmission capacity are likely to change the physical characteristics (e.g. impedances) of the existing network, raising the issue of investment externality", but fails to go further into the problem.

Is the addition or removal of transmission circuits necessary for markets to fail? In other words, if we only allow investment that results in an upgrade of the line capacities of a given grid, will a competitive equilibrium allocation fail to be efficient? Are the externalities created by loop flows due to the fact that the allowable injections at one node depend on the injections at the other nodes? Or are they related to the fact that changes in line capacities affect the set of feasible injections into the grid? Does the existence of loop flows result in externalities for sure? These questions need to be answered in order to understand how loop flows create externalities that cause the market failure with endogenous transmission investment. In this paper, using a partial equilibrium approach, we clarify the nature of the externalities associated with loop flows that cause transmission investment to be inefficient. The bottom line is that transmission investment introduces an externality only if it affects the flow of power along the lines for any given set of net injections. For instance, the addition or removal of a new circuit will affect the flow of power for any given set of net injections, unless of course we are adding or removing part of a radial network. But the increase of the operational capacity of a line will not introduce an externality, even if it does change the set of feasible injections, unless it also affects the flow of power for any given set of injections.

From the engineering perspective, there are two options for expanding transmission: (1) build new transmission circuits or upgrade old ones and (2) introduce additional control capability. Investments in transmission expansion include building new transmission circuits, upgrading old and introduce additional control capability. In this paper, we consider transmission investments in both options. Although both will continue to exist as options, (1) has and will become less and less viable. As a result, there is significantly increased potential

[3] A radial network refers to a network with no closed loop.

for application of additional power system control in order to strengthen and expand transmission in the face of growing transmission usage. However, there has been little effort towards planning transmission in the sense of this option, yet the ability to consider it in the planning process is a clear need to the industry. In this paper we make steps forward in the economic analysis of placing control.

The rest of this chapter is organized as follows. Section 2.2 is a brief introduction of physics fundamentals in power transmission. In section 2.3, we formulate and solve the general model with endogenous transmission investment for a fixed grid topology. Sections 2.4 and 2.5, each give an analytical example of transmission investments in building new lines and in placing control, respectively, as an application of the model in section 2.3. Grid expansion is studied through a numerical example in section 2.6. The conclusions and implications are summarized in the last section.

2.2 Basics of Power Flow Model

This section addresses some technical issues related to electric power transmission, which are part of our economic model. We employ a simplified power system model as an approximation to the AC (alternate current) system, leaving out some aspects such as reactive power and line losses.

2.2.1 Real Power Flow

Every AC electrical network has both real and reactive power flows. The sinusoidal pattern of instantaneous power flow produces a complex power representation with real and imaginary parts that correspond to real and reactive power, respectively. In this paper, we only consider real power.

Consider a transmission network consisting of a set of nodes or buses $N = \{1,...,N\}$ and a set of links $L = \{1,...,L\}$. Each link $\ell \in L$ represents a transmission line connecting two nodes in N through which power can flow[4]. Not all pairs of nodes need to be connected transmission lines. A line connecting nodes i and j is characterized by its impedance denoted

[4] By impedance, we mean reactance, the imaginary part of impedance. There may be more than one line between a certain pair of nodes. Here we treat them as one line or, by another name one corridor connecting the two nodes.

by Z_{ij} [5]. We have $Z_{ij} = Z_{ji}$. For each node $n \in N$, let x_n denote the net power injection at *n*. In a lossless network, the power injections add up to zero: $\sum_{n=1}^{N} x_n = 0$, and thus knowledge of the net injections in all but one of the nodes is enough to know the net injection in the remaining node. In what follows, we will choose node *N* to be the residual node (or reference node), and express the dispatches in terms of the other *N*-1 nodes. For each pair of connected nodes (i, j), let $f_{i \to j}$ be the flow from *i* to *j* through the connecting line. Appendix 1 shows that under some certain assumptions, these flows can be written as a linear combination of the net injections. Specifically, there are unique coefficients α_n^{ij}, $n \in N$, such that for any dispatch $(x_1, ..., x_N)$ the flow from *i* to *j* can be written as:

$$f_{i \to j} = \sum_{n=1}^{N-1} \alpha_n^{ij} x_n, \; i, j = 1, ..., N \tag{1}$$

The coefficients α_n^{ij} are known as "distribution" or "shift" factors, interpreted as the proportion of the power injected at node *n* that goes from *i* to *j* through the connecting line, given node *N* as the reference node. Note that depending on the dispatch $(x_1, ..., x_N)$, $f_{i \to j}$ can be positive or negative, and that since $-f_{i \to j} = f_{j \to i} = \alpha_n^{ji} x_n$, we have $\alpha_n^{ji} = -\alpha_n^{ij}$. We will sometimes want to talk about the flow along line ℓ connecting nodes *i* and *j*, without specifying the direction. This flow will be denoted $f_\ell = \alpha_n^\ell x_n = \max\left\{ \sum_{n=1}^{N-1} \alpha_n^{ij} x_n, \sum_{n=1}^{N-1} \alpha_n^{ji} x_n \right\}$. Accordingly, α_n^ℓ is the proportion of each unit of power injected in *n* that goes in the "positive" direction of line ℓ.

In general, the values of the distribution factors depend on the impedances. Given a grid, α_n^{ij} are known constants, since Z_{ij} are fixed. For a radial network, however, the distribution factors only take the values of 0, 1 or -1, independent of the line impedances. That is, for each unit of power injected at one node and withdrawn at another, the lines that it

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[5] If there are several lines joining *i* and *j*, Z_{ij}, the total impedance of the corridor *ij* equals one over the sum of the reciprocals of the individual lines' impedances. If there is no such line, $Z_{ij} = \infty$.

transits have distribution factors 1 or -1 and the lines it does not go through have the distribution factor 0.

2.2.2 Flow Constraints

The line flows need to satisfy several constraints. Each transmission line has a maximum acceptable flow, called capacity. It is usually determined by the minimum of the thermal limit, voltage limit and stability limit of the line. Exceeding the capacity can cause physical damage to the transmission line, with subsequent high probability of power failure. So the following capacity constraint must be satisfied for each line

$$f_\ell \leq k_\ell \tag{2}$$

Where k_ℓ is line ℓ's capacity.

Another set of constraints are called contingency constraints. Sometimes, one or more of the transmission lines may be out of work in a contingency. This changes the network and leads to a new set of line flows that may no longer meet the capacity limits in (2). For operational security, additional restrictions are imposed on the pre-contingency line flows so that the post-contingency network flows also satisfy the capacity constraints. These additional constraints are nothing more than capacity constraints in a contingency, but they constrain the network all the time, not only when a contingency does occur. We will ignore contingency constraints in sections 3 and 4 and consider them in section 5.

2.3 The Model

In this section, we set up a multi-node power transmission model, using the partial equilibrium competitive analysis. In this model, transmission investment is endogenized. The purpose is to see if decentralized transmission investment is efficient and why or why not. We shall restrict attention to a fixed grid topology, and will comment on the expansion of the grid later.

2.3.1 Model Specification and Assumptions

There are two commodities in the economy: power and the numeraire[6]. The original transmission grid consists of N nodes, indexed $n = 1,...,N$. Some pairs of nodes are connected by transmission lines, and others are not. Let L be the set of existing corridors. In the beginning, each corridor, ℓ, has a capacity limit $k_\ell^0 > 0$ and an impedance Z_ℓ^0. For simplicity, we assume that all the existing lines of the network are owned by a single transmission owner (TO).

There is a set E of transmission investment firms indexed by $r \in E$. Through investment, they increase the transmission capacities of the different corridors, using the numeraire as the input. Each investment firm's technology is given by a production set

$$Y^r = \left\{\left(-z^r, I_1^r,...,I_L^r\right): I_\ell^r \geq 0\ \forall \ell \in L \text{ and } z^r \geq C^r\left(I_1^r,...,I_L^r\right)\right\},\ r \in E \qquad (3)$$

where $C^r(\cdot): R_+^L \to R_+$ is investment firm r's cost function, which is assumed to be twice differentiable and convex, with $\nabla C^r(\cdot) > 0$ and $C^r(0) = 0$. I_ℓ^r is the extra transmission capacity on corridor ℓ created by transmission investment firm r, and z^r is the amount of numeraire required as input. The total investment on corridor ℓ is $I_\ell = \sum_{r \in E} I_\ell^r$. As a result of the investment, the total capacity of corridor ℓ becomes $k_\ell = k_\ell^0 + I_\ell$. Investment, while enhancing the line capacities, may change the impedances at the same time and in turn affect the distribution factors[7]. We will denote the distribution factors by $\alpha_n^\ell\left((I_v)_{v \in L}\right)$ to stress their dependence on investment.

Without loss of generality, assume that at each node there is only one consumer and one generator, both indexed by the node, $n = 1,...,N$ where they are located. Each generator produces power using the numeraire according to the following technology:

$$Y_n = \left\{\left(-z_n, q_n\right): q_n \geq 0 \text{ and } z_n \geq C_n(q_n)\right\},\ n = 1,...,N$$

[6] Transmission is not directly consumed by the consumers, so we do not view it as a commodity, although there is market for it.

[7] Usually when the capacity of a line is enhanced, its impedance will be lower.

where $C_n(q_n)$ is generator n's cost function, which is assumed to be twice differentiable, strictly increasing and concave, and satisfies $C_n(0)=0$, z_n is the amount of numeraire used for production, and q_n is the amount of power generated[8].

Each consumer is endowed with a fixed amount $\omega_n > 0$ of the numeraire and has a quasi-linear utility function $u_n : R \times R_+ \to R$

$$u_n(m_n, c_n) = m_n + \phi_n(c_n),\ n = 1,\dots,N$$

where $c_n \geq 0$ and $m_n \in R$ are consumer n's consumption of power and of the numeraire, respectively. As usual, $\phi_n(c_n)$ is assumed to be bounded above, twice differentiable, strictly increasing and strictly concave. The total resources of the economy consist of the aggregate endowment of the numeraire, $\varpi = \sum_{n=1}^{N} \omega_n$. Assume that the consumer at node n owns a share θ_n^j of the generator located at node j, a share θ_n^T of the TO and a share θ_n^r of investment firm r. Clearly, $\sum_{n=1}^{N} \theta_n^j = 1$ for all j = *1,…,N*, $\sum_{n=1}^{N} \theta_n^r = 1$ for all $r \in E$, and $\sum_{n=1}^{N} \theta_n^T = 1$. The shares θ_n^T and θ_n^r are closely related to what is referred to in the literature as physical transmission rights.

A *dispatch* is a vector $x = (x_1,\dots,x_N) \in R^N$ of net power injections to the grid, one for each node such that $\sum_{n=1}^{N} x_n = 0$. A dispatch is *feasible* if it satisfies the following flow restrictions, as given in section 2:

$$f_\ell(x) = \sum_{n=1}^{N-1} \alpha_n^\ell\left((I_v)_{v\in L}\right) x_n \leq k_\ell^0 + I_\ell,\ \ell \in L \tag{4}$$

2.3.2 Efficient Allocation

An *allocation* in this economy is a description of each consumer's consumption plan, each generator's production plan and each investment firm's investment plan. Some

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[8] In the engineering literature, *p*, instead of *q* is usually used for the quantity of power. In our paper, we maintain the economics convention, using *q* as the quantity and reserving *p* for the price.

allocations are not feasible. In order to be feasible, an allocation must satisfy the constraints imposed by the economy's resources, technology and grid capacity. Formally,

Definition 1 A feasible allocation $\left((m_n, c_n)_{n=1}^N, (-z_n, q_n)_{n=1}^N, \left(-z^r, (I_\ell^r)_{\ell \in L}\right)_{r \in E}\right)$ in this economy is a specification of a consumption bundle $(m_n, c_n) \in R \times R_+$ for each consumer *n* = *1,…,N*, a production plan $(-z_n, q_n) \in Y_n$ for each generator *n* = *1,…,N* and an investment plan $\left(-z^r, (I_\ell^r)_{\ell \in L}\right) \in Y_r$ for each investment firm $r \in E$, such that

$$\sum_{n=1}^{N} m_n + \sum_{n=1}^{N} z_n + \sum_{r \in E} z^r = \varpi \tag{5}$$

$$\sum_{n=1}^{N} c_n = \sum_{n=1}^{N} q_n \tag{6}$$

$$\sum_{n=1}^{N-1} \alpha_n^\ell \left((I_v)_{v \in L}\right)(q_n - c_n) \le k_\ell^0 + I_\ell, \ \forall \ell \in L \tag{7}$$

Condition (5) requires that the total amount of the numeraire consumed and used for production and investment should be equal to the amount of the numeraire that is available to society. Condition (6) dictates that the total generation of the system should satisfy the aggregate demand. Condition (7) requires that the flow along each line should satisfy the capacity constraint.

Although there can be many feasible allocations, not all of them are equally attractive. We are interested in those feasible allocations that cannot be improved upon.

Definition 2 A feasible allocation $\left((m_n^*, c_n^*)_{n=1}^N, (-z_n^*, q_n^*)_{n=1}^N, \left(-z^{r*}, (I_\ell^{r*})_{\ell \in L}\right)_{r \in E}\right)$ is efficient (or optimal) if there is no alternative feasible allocation $\left((m_n, c_n)_{n=1}^N, (-z_n, q_n)_{n=1}^N, \left(-z^r, (I_\ell^r)_{\ell \in L}\right)_{r \in E}\right)$ such that

$$u_n(m_n, c_n) \ge u_n(m_n^*, c_n^*), \ \forall n = 1, \ldots, N$$

with strict inequality for at least one agent *n*.

This definition states that when the economy is at an efficient allocation, no other feasible allocation can make at least one consumer better-off without hurting any of the other agents.

2.3.3 Characterization of Efficient Allocation

When consumer preferences are quasi-linear, the optimal allocation must maximize the sum of individual utilities. Consider an allocation $\left((m_n, c_n)_{n=1}^N, (-z_n, q_n)_{n=1}^N, \left(-z^r, (I_\ell^r)_{\ell\in L}\right)_{r\in E}\right)$. The total amount of numeraire used as inputs for producing power and transmission is $\sum_{n=1}^N C_n(q_n) + \sum_{r\in E} C^r\left(I_1^r,...,I_L^r\right)$. The leftover to distribute among the consumers is $\varpi - \sum_{n=1}^N C_n(q_n) - \sum_{r\in E} C^r\left(I_1^r,...,I_L^r\right)$. Therefore, given the quasi-linearity of the consumers' preferences, the sum of the utilities can be written as $\sum_{n=1}^N \phi_n(c_n) - \sum_{n=1}^N C_n(q_n) - \sum_{r\in E} C^r\left(I_1^r,...,I_L^r\right) + \varpi$. An optimal allocation $\left((m_n^*, c_n^*)_{n=1}^N, (-z_n^*, q_n^*)_{n=1}^N, \left(-z^{r*}, (I_\ell^{r*})_{\ell\in L}\right)_{r\in E}\right)$ therefore solves

$$\max_{\substack{c_n, q_n, I_\ell^r \geq 0 \\ n\in N, \ell\in L, r\in E}} \sum_{n=1}^N \phi_n(c_n) - \sum_{n=1}^N C_n(q_n) - \sum_{r\in E} C^r\left(I_1^r,...,I_L^r\right) \tag{8}$$

$$\text{s.t.} \sum_{n=1}^N c_n = \sum_{n=1}^N q_n$$

$$\sum_{n=1}^{N-1} \alpha_n^\ell\left((I_v)_{v\in L}\right)(q_n - c_n) \leq k_\ell^0 + I_\ell, \ \forall \ell \in L$$

That is, an optimal allocation maximizes the aggregate surplus subject to the non-negativity, balancing and flow constraints. For the sake of analysis, assume that $\alpha_n^\ell(\cdot)$ are convex functions so that the set of feasible allocations is convex. Let λ and μ_ℓ be the Lagrangian multipliers of the above constraints, respectively. Then the first order conditions for an efficient allocation are

$$\frac{\partial \phi_n}{\partial c_n}(c_n^*) \leq \lambda - \sum_{\ell\in L} \alpha_n^\ell\left((I_v^*)_{v\in L}\right)\mu_\ell, \text{ with equality if } c_n^* > 0, \ \forall n = 1,..., N-1$$

$$\frac{\partial \phi_N}{\partial c_N}(c_N^*) \leq \lambda, \text{ with equality if } c_N^* > 0$$

$$\lambda - \sum_{\ell\in L} \alpha_n^\ell\left((I_v^*)_{v\in L}\right)\mu_\ell \leq \frac{\partial C_n}{\partial q_n}(q_n^*), \text{ with equality if } q_n^* > 0, \ \forall n = 1,..., N-1$$

$$\lambda \leq \frac{\partial C_N}{\partial q_N}\left(q_N^*\right), \text{ with equality if } q_N^* > 0$$

$$\mu_\ell - \sum_{z\in L}\mu_z \sum_{n=1}^{N-1} \frac{\partial \alpha_n^z\left(\left(I_v^*\right)_{v\in L}\right)}{\partial I_\ell}\left(q_n^* - c_n^*\right) \leq \frac{\partial C^r}{\partial I_\ell^r}\left(I_v^*\right)_{v\in L}, \text{ with equality if } I_\ell^* > 0,\ \forall \ell \in L, r \in E$$

$$\sum_{n=1}^{N} c_n^* = \sum_{n=1}^{N} q_n^*$$

$$\sum_{n=1}^{N-1} \alpha_n^\ell\left(\left(I_v^*\right)_{v\in L}\right)\left(q_n^* - c_n^*\right) \leq k_\ell^0 + I_\ell^*, \text{ with equality if } \mu_\ell > 0,\ \forall \ell \in L$$

Let $\left(\left(m_n^*, c_n^*\right)_{n=1}^N, \left(-z_n^*, q_n^*\right)_{n=1}^N, \left(-z^{r*}, \left(I_\ell^{r*}\right)_{\ell\in L}\right)_{r\in E}\right)$ be an efficient allocation. The last two equations are the feasibility conditions: aggregate consumption should be equal to aggregate power generation, and the resulting dispatch should induce line flows that respect the corresponding capacity constraints. The multipliers μ_ℓ, for $\ell \in L$ are the marginal social benefit that would result from an increase of one MW in the transmission capacity of line ℓ. Equivalently, they are the marginal social cot of having one MW less of capacity available on line ℓ. Only if the capacity constraint is binding at the efficient allocation, can this marginal social benefit be positive. The multiplier λ is the marginal social benefit of one MW consumed at the residual node *N*, or equivalently the marginal social cost of one MW produced at node *N*.

The first two sets of conditions require that the private benefit from an additional unit of power supplied at node *n*, for *n* = *1,…,N*, be equal to the social cost of supplying it at that node, unless $c_n^* = 0$, in which case the private benefit can be lower than the social cost. Here the social cost of supplying one MW at node *n* equals the social cost of supplying one MW at the residual node, λ, plus the social cost of transmitting it to node *n*, which is given by $-\sum_{\ell\in L}\alpha_n^\ell\left(\left(I_v^*\right)_{v\in L}\right)\mu_\ell$.

To see this, note that $\alpha_n^\ell\left(\left(I_v^*\right)_{v\in L}\right)$ is the fraction of each MW *injected* at node *n* that goes through line ℓ in the "positive" direction. The social cost of transmitting this fraction along this line is $\alpha_n^\ell\left(\left(I_v^*\right)_{v\in L}\right)\mu_\ell$. As a result, the cost of transmitting one MW from node *n* to node N is $\sum_{\ell \in L} \alpha_n^{\ell}\left(\left(I_v^*\right)_{v\in L}\right)\mu_\ell$. Consequently, the social cost of transmitting one MW in the opposite direction, namely from N to n, is $-\sum_{\ell \in L} \alpha_n^{\ell}\left(\left(I_v^*\right)_{v\in L}\right)\mu_\ell$.

The second two sets of equations necessitate that the private cost of generation of one MW at each node n, should be equal to the social cost of an additional MW at that node, unless $q_n^* = 0$, in which case the private cost can be greater than the social cost. As explained earlier, the social cost of an additional MW at node n is $\lambda - \sum_{\ell \in L} \alpha_n^{\ell}\left(\left(I_v^*\right)_{v\in L}\right)\mu_\ell$.

Lastly, the third set of equations demand that the marginal private costs of investing in capacity of line ℓ be equal to the social benefit of that investment, unless $I_\ell^* = 0$, in which case the private cost of the investment can be greater than the social benefit. This social benefit has two components. One is the social benefit of the increased capacity of the line, which is μ_ℓ. The other is the social cost that results from the change in the distribution factors that is caused by the investment on line ℓ. To calculate this social cost, note that the investment causes the distribution factors to change at a rate of $\frac{\partial \alpha_n^z\left(\left(I_v^*\right)_{v\in L}\right)}{\partial I_\ell}$. Given that the injection at node n is $q_n^* - c_n^*$, this means that the flow along line z due to this injection increases by $\frac{\partial \alpha_n^z\left(\left(I_v^*\right)_{v\in L}\right)}{\partial I_\ell}\left(q_n^* - c_n^*\right)$, which amounts to saying that the given injection requires more of the capacity of line ℓ. Therefore, the social cost of this required capacity is $\mu_z \frac{\partial \alpha_n^z\left(\left(I_v^*\right)_{v\in L}\right)}{\partial I_\ell}\left(q_n^* - c_n^*\right)$. As a result, the total social cost that results from the change in the distribution factors induced by the investment in line ℓ is given by $\sum_{z\in L} \mu_z \sum_{n=1}^{N-1} \frac{\partial \alpha_n^z\left(\left(I_v^*\right)_{v\in L}\right)}{\partial I_\ell}\left(q_n^* - c_n^*\right)$.

2.3.4 Competitive Equilibrium

Now that we have a characterization of the efficient allocation, we can ask how to implement it. One alternative would be to impose it by a central planner. This entity knows what the efficient allocation is and can, in principle, dictate the optimal generation levels to the generators and the optimal capacitor-induced capacity enhancement to the investment firm. In reality, however, trying to impose an allocation on the different players may be an impossible task. One would have to know the cost structure of every generator and of the investment firms, and more importantly, one would have to possess the power to impose on them the optimal generation and investment levels. Another alternative would be to decentralize the decisions by means of a price system and competitive markets. The idea of such a price system is to allow the generators and investment firms to decide for themselves the generation and investment levels, respectively, taking electricity prices and transmission charges as given. The objective is still the same, but the huge task of determining the optimal allocation is now subdivided into many small tasks, each performed by an economic agent. Nobody needs to know the technology or cost structure of all the firms. It is enough for each firm to know its own cost function. Similarly, it is not needed for any omniscient central planner to figure out the optimal allocation. Each agent will try to maximize her own profit or utility given the market prices. Presumably, the agents will decide what is best for them, but if the prices are right, these prices will induce the agents to choose the quantities that correspond to the efficient allocation.

We now describe a competitive equilibrium. In the equilibrium, there will be electricity prices p_n associated with each node (the *nodal* prices) $n = 1,\dots,N$ and transmission price t_ℓ for each corridor $\ell \in L$. The price of the numeraire is normalized to unity. Each generator decides how much electricity to produce in response to its own nodal price and each consumer decides how much electricity to consume in response to her own nodal price. Given all the nodal prices and transmission prices, each investment firm chooses how much extra capacity to build through transmission investment. After the investment, the transmission investment firms become the owners of the newly produced capacities. In order for the nodal and transmission prices $\left\langle (p_n)_{n=1}^N, (t_\ell)_{\ell\in L} \right\rangle$ to be in equilibrium, they must satisfy a no-arbitrage condition. Specifically, it must be impossible for any individual to make a positive profit by buying power at a given node and selling it at a different node at the corresponding nodal prices after paying the transmission charge induced by this transaction. We define a competitive equilibrium as follows.

Definition 3 An allocation $\left((m_n^*, c_n^*)_{n=1}^N, (-z_n^*, q_n^*)_{n=1}^N, \left(-z^{r*}, (I_\ell^{r*})_{\ell\in L}\right)_{r\in E}\right)$ and a price vector $(p,t) = \left((p_n)_{n=1}^N, (t_\ell)_{\ell\in L}\right)$ constitute a competitive equilibrium if the following conditions are satisfied:

1. Generators' profit maximization: For each generator $n = 1, ..., N$, $(-z_n^*, q_n^*) \in Y_n$ satisfies

$$p_n q_n^* - z_n^* \geq p_n q_n - z_n, \ \forall (-z_n, q_n) \in Y_n$$

2. Investment firms' profit maximization: For each investment firm $r \in E$, $\left(-z^{r*}, (I_\ell^{r*})_{\ell\in L}\right) \in Y^r$ satisfies

$$\sum_{\ell\in L} t_\ell I_\ell^{r*} - z^{r*} \geq \sum_{\ell\in L} t_\ell I_\ell^r - z^r, \ \forall \left(-z^r, (I_\ell^r)_{\ell\in L}\right) \in Y^r$$

3. Utility maximization: For each consumer $n = 1, ..., N$, (m_n^*, c_n^*) solves

$$\max_{(m_n, c_n)\in R\times R_+} m_n + \phi_n(c_n)$$

$$\text{s.t. } m_n + p_n c_n \leq \omega_n + \sum_{j=1}^N \theta_n^j (p_j q_j^* - z_j^*) + \sum_{r\in E} \theta_n^r \left(\sum_{\ell\in L} t_\ell I_\ell^{r*} - z^{r*}\right) + \theta_n^T \sum_{\ell\in L} t_\ell k_\ell^0$$

4. No arbitrage: for any dispatch $(x_1, ..., x_N) \in R^N$ such that $\sum_{n=1}^N x_n = 0$

$$\sum_{n=1}^N p_n x_n + \sum_{\ell\in L} t_\ell \sum_{n=1}^N \alpha_n^\ell \left((I_v^*)_{v\in L}\right) x_n \geq 0 \tag{9}$$

5. Market clearing:

$$\sum_{n=1}^N m_n^* + \sum_{n=1}^N z_n^* + \sum_{z\in E} z^{r*} = \varpi \tag{10}$$

$$\sum_{n=1}^N c_n^* = \sum_{n=1}^N q_n^* \tag{11}$$

$$\sum_{n=1}^{N-1} \alpha_n^\ell \left((I_v^*)_{v\in L}\right)(q_n^* - c_n^*) \leq k_\ell^0 + I_\ell^*, \text{ with equality if } t_\ell > 0, \ \forall \ell \in L \tag{12}$$

The first two conditions state that generators and transmission investment firms choose production and investment plans that maximize their profits, given the competitive prices. The third condition states that consumers maximize their profits given their budget constraints. Condition 4 is the no-arbitrage condition; it should be impossible to find a dispatch that yields positive profits. Condition 5 dictates that in a competitive equilibrium market must clear for each good: the numeraire, power and transmission, respectively. (11) actually requires that the total amount of power consumed is equal to the total amount of power produced. (12) says that in equilibrium the price of transmission on a line is positive only when the demand for transmission equals the supply of transmission on that line[9]. Alternatively, the transmission price is zero when there is excess capacity.

2.3.5 Characterization of Competitive Equilibrium

Now let us characterize the five conditions in the above definition. Condition 1 says that each generator maximizes its profit, given its own technology and nodal price: formally, it chooses q_n^*, so as to solve:

$$\max_{q_n \geq 0} p_n q_n - C_n(q_n)$$

Given our assumptions on the cost function, the necessary and sufficient conditions for q_n^* to solve the above problem are

$$p_n \leq \frac{\partial C_n(q_n^*)}{\partial q_n}, \text{ with equality if } q_n^* > 0, \forall n = 1,\ldots,N \tag{13}$$

In condition 2, each investment firm maximizes its profit, taking as given the transmission price on each line and its own technology: it chooses $(I_1^{r*},\ldots,I_L^{r*})$ which solves:

$$\max_{I_\ell^r, \forall \ell \in L} \sum_{\ell \in L} t_\ell I_\ell^r - C^r(I_1^r,\ldots,I_L^r)$$

The corresponding necessary and sufficient conditions are

$$t_\ell \leq \frac{\partial C^r}{\partial I_\ell^r}(I_1^{r*},\ldots,I_\ell^{r*}), \text{ with equality if } I_\ell^{r*} > 0, \forall r \in E \tag{14}$$

Condition 3 in the definition states that each consumer maximizes her utility, in response to the nodal price at her own node: she solves the problem

[9] That is, when the line is congested.

$$\max_{(m_n, c_n) \in R \times R_+} m_n + \phi_n(c_n)$$

$$\text{s.t. } m_n + p_n c_n \leq w_n$$

where $w_n = \omega_n + \sum_{j=1}^{N} \theta_n^j \left(p_n q_j^* - C_j\left(q_j^*\right)\right) + \sum_{r \in E} \theta_n^r \left(\sum_{\ell \in L} t_\ell I_\ell^{r*} - C^r\left(I_\ell^{r*}\right)\right) + \theta_n^T \sum_{\ell \in L} t_\ell k_\ell^0$ is consumer n's wealth. Given our assumptions about the utility functions, the necessary and sufficient conditions for a utility maximizing bundle are

$$\frac{\partial \phi_n\left(c_n^*\right)}{\partial c_n} \leq p_n, \text{ with equality if } c_n^* > 0, \forall n = 1,...,N \quad (15)$$

As for the no-arbitrage condition 4, we will show that a necessary and sufficient condition for it to hold is that the price at the residual node N be equal to the nodal price at each node $n=1,\ldots,N-1$ plus the charge that results from the transmission of one unit of power from node n to node N. Formally, condition (9) is equivalent to

$$p_N = p_n + \sum_{\ell \in L} t_\ell \alpha_n^\ell\left(\left(I_v^*\right)_{v \in L}\right), \ \forall n = 1,..., N-1 \quad (16)$$

To see that this condition is necessary, note that (9) should be satisfied with equality, since if $(x_1,...,x_N)$ is a feasible dispatch, so is $(-x_1,...,-x_N)$. Therefore,

$$\sum_{n=1}^{N} p_n x_n + \sum_{\ell \in L} t_\ell \alpha_n^\ell\left(\left(I_v^*\right)_{v \in L}\right) x_n = 0$$

Taking $x_N = 1$, $x_m = -1$, and $x_k = 0$, for $k \neq n, N$, and rearranging it we get condition (16). To see that this condition is also sufficient, note that if (16) holds for $n=1,\ldots,N-1$, then we have that for any feasible dispatch $(x_1,...,x_N)$

$$p_N x_n = p_n x_n + \sum_{\ell \in L} t_\ell \alpha_n^\ell\left(\left(I_v^*\right)_{v \in L}\right) x_n, \ \forall n = 1,..., N-1$$

Adding over $n = 1,..., N-1$ we get

$$\sum_{n=1}^{N-1} p_N x_n = \sum_{n=1}^{N-1} p_n x_n + \sum_{n=1}^{N-1} \sum_{\ell \in L} t_\ell \alpha_n^\ell\left(\left(I_v^*\right)_{v \in L}\right) x_n$$

which, given that $x_N = -\sum_{n=1}^{N-1} x_n$ implies the no arbitrage condition (9).

The last condition says that at the equilibrium prices, supply equals demand for the numeraire, power and transmission, respectively. In all, conditions (10) through (15) must be satisfied in a competitive equilibrium.

To summarize, the necessary and sufficient conditions for allocation $\left((m_n^*, c_n^*)_{n=1}^N, (-z_n^*, q_n^*)_{n=1}^N, (-z^{r*}, (I_\ell^{r*})_{\ell\in L})_{r\in E}\right)$ and price vector $(p,t) = \left((p_n)_{n=1}^N, (t_\ell)_{\ell\in L}\right)$ to be a competitive equilibrium are

$$p_n \le \frac{\partial C_n(q_n^*)}{\partial q_n}, \text{ with equality if } q_n^* > 0, \forall n = 1,\ldots,N \tag{17}$$

$$t_\ell \le \frac{\partial C^r}{\partial I_\ell^r}(I_1^{r*},\ldots,I_\ell^{r*}), \text{ with equality if } I_\ell^{r*} > 0, \forall r \in E \tag{18}$$

$$\frac{\partial \phi_n(c_n^*)}{\partial c_n} \le p_n, \text{ with equality if } c_n^* > 0, \forall n = 1,\ldots,N \tag{19}$$

$$p_N = p_n + \sum_{\ell\in L} t_\ell \alpha_n^\ell\left((I_v^*)_{v\in L}\right), \ \forall n = 1,\ldots,N-1 \tag{20}$$

$$\sum_{n=1}^N m_n^* + \sum_{n=1}^N z_n^* + \sum_{z\in E} z^{r*} = \varpi \tag{21}$$

$$\sum_{n=1}^N c_n^* = \sum_{n=1}^N q_n^* \tag{22}$$

$$\sum_{n=1}^{N-1} \alpha_n^\ell\left((I_v^*)_{v\in L}\right)(q_n^* - c_n^*) \le k_\ell^0 + I_\ell^*, \text{ with equality if } t_\ell > 0, \ \forall \ell \in L \tag{23}$$

2.3.6 Comparison between Competitive Equilibrium and Efficient Allocation

So far, we have derived the conditions for both the competitive equilibrium and efficient allocation. Now comes the question we are interested in: is the market outcome socially efficient? Or equivalently, can the efficient allocation be decentralized? To answer the question, we need to compare the two sets of conditions.

The main observation we can make is that if for each lines, $\ell, \ell' \in L$, and for each node $n \in N$, investment in line ℓ does not affect the distribution factor $\alpha_n^{\ell'}$, that is, if

$\frac{\partial \alpha_n^z\left(\left(I_v^*\right)_{v\in L}\right)}{\partial I_\ell}=0$, then every efficient allocation can be decentralized by competitive prices, and every competitive allocation is efficient. Indeed, it is easy to see that if λ and $\left(\mu_\ell\right)_{\ell\in L}$ are the Lagrangian multipliers that, together with $\left(\left(c_n^*,q_n^*\right)_{n=1}^N,\left(I_\ell^{r*}\right)_{r\in E,\ell\in L,}\right)$ solve the necessary and sufficient conditions for an efficient allocation, then $p_N=\lambda$, $p_n=\lambda-\sum_{\ell\in L}\alpha_n^\ell\mu_\ell$ for $n=1,...,N-1$, and $t_\ell=\mu_\ell$, for $\ell\in L$ are the nodal and transmission prices that support the corresponding efficient allocation. And conversely, if $\left(\left(p_n\right)_{n=1}^N,\left(t_\ell\right)_{\ell\in L}\right)$ are competitive equilibrium prices that together with $\left(\left(c_n^*,q_n^*\right)_{n=1}^N,\left(I_\ell^{r*}\right)_{r\in E,\ell\in L,}\right)$ solve the necessary and sufficient conditions for a competitive equilibrium, then $\lambda=p_N$, and $\mu_\ell=t_\ell$, together with $\left(\left(c_n^*,q_n^*\right)_{n=1}^N,\left(I_\ell^{r*}\right)_{r\in E,\ell\in L,}\right)$ satisfy the necessary and sufficient conditions for an efficient allocation.

In general, however, the distribution factors $\alpha_n^{\ell'}$ are affected by the investment in the different lines. That is, transmission investment usually changes the flow of power along the lines for a given set of injections. This externality is the root cause of the inefficiency of the competitive allocations, and of the fact that efficient allocation cannot be decentralized by means of competitive prices. Suppose the transmission investment in line ℓ enhances $\alpha_n^{\ell'}$ for some line $\ell'\in L$. Without loss of generality, let us assume that $\alpha_n^{\ell'}>0$. Then for each unit of power injected at *n*, a larger proportion will transit line ℓ' as a result of the investment. Recall that x_n, the net power injection at node *n* can be positive or negative. Someone who injects x_n at *n* will have to pay $\alpha_n^{\ell'}x_n$ times the corresponding transmission price for using line ℓ'. Suppose that line ℓ' is congested, so that its transmission price is positive. Then if $x_n>0$, the injector will need to make a positive payment; if $x_n<0$, the injector (or actually the ejector in this case) will pay a negative amount. Since $\alpha_n^{\ell'}$ increases after the investment, the congestion payment will be higher than before for $x_n>0$ and lower than before for $x_n<0$. So, the transmission investment creates a negative externality towards those who inject power at node n and a positive externality for those who eject power at n. Clearly, this externality is relevant only if the line is congested. The transmission investment in one line ℓ may change the distribution factors of several lines in the network and have an externality on each of the affected lines. The sum of the values of all these externalities is the total effect caused by the investment on line ℓ, indicated by the expression $\sum_{z\in L}\mu_z\sum_{n=1}^{N-1}\frac{\partial\alpha_n^z\left(\left(I_v^*\right)_{v\in L}\right)}{\partial I_\ell}\left(q_n^*-c_n^*\right)$. As this effect is not taken into account by the competitive prices, one would expect competitive markets to result in under- or over-investment in transmission. One way to restore efficiency is to impose a unit tax of $\tau_\ell=\sum_{z\in L}\mu_z\sum_{n=1}^{N-1}\frac{\partial\alpha_n^z\left(\left(I_v^*\right)_{v\in L}\right)}{\partial I_\ell}\left(q_n^*-c_n^*\right)$ or equivalently a subsidy of $s_\ell=-\sum_{z\in L}\mu_z\sum_{n=1}^{N-1}\frac{\partial\alpha_n^z\left(\left(I_v^*\right)_{v\in L}\right)}{\partial I_\ell}\left(q_n^*-c_n^*\right)$ on investment in line ℓ.

Recall that in a radial network the distribution factors are constants of 1, -1 or 0, independent of the line impedances. As long as the radial grid topology is not changed by the transmission investment, the distribution factors will remain the same. And there will be no investment externality. Consider the network in figure 1. If new transmission lines are built between nodes 1 and 3 or between 2 and 3, the grid topology will not change and the distribution factors will be the same as before regardless of the newly created capacities. In that case, $\frac{\partial\alpha_n^z\left(\left(I_v^*\right)_{v\in L}\right)}{\partial I_\ell}=0$, for all $\ell, z=1,2$ and $n=1,2,3$ and the investment in equilibrium will be efficient. If new lines are built between nodes 1 and 2, the grid topology will be changed and the network will become a meshed one. We will leave this case for discussion in the following subsection.

Now it is clear that the externalities that cause endogenous transmission investment to be inefficient are created by loop flows and there is no externality with radial networks. But, the existence of loop flows does not necessarily cause externalities. It does only when transmission investment changes the distribution factors, thus affects the flow of power on the lines for a given set of injections. Otherwise, even in the presence of loop flows, there will be no externalities and the investment will be efficient.

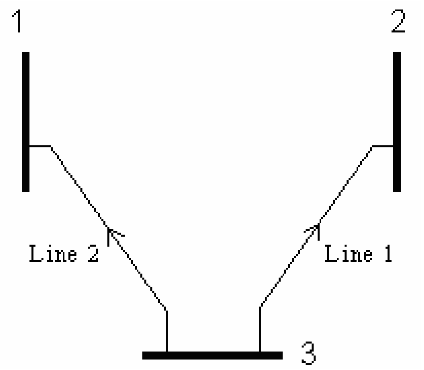

Figure 1: A radial network

In conclusion, for endogenous transmission investment, the source of the market failure lies in the fact that investment affects the power flow through the lines for any given dispatch. Efficiency is never guaranteed as long as transmission investment changes the distribution factors, whether in the case of a fixed grid topology or grid expansion. There is no externality with a radial network if it is still radial after the investment. Interestingly, here the action of an investment firm does not directly affect the other investment firms' production possibility sets, the generators' production sets or the consumers' preferences. Instead, the investment changes the flow structure of the transmission network, which causes inefficiency. Therefore, the externality introduced by transmission investment is different from the externality in the usual sense. It arises due to the physical aspects in power transmission.

For application of the model, we will, in the following sections, give two analytical examples, one about the capacity enhancement via lines and the other about the capacity enhancement via control. The comparison of the two examples makes it even clearer when transmission investment introduces an externality, hence results in market failure and when it does not.

2.4 Transmission-induced Capacity Enhancement

The preceding section formulates the general model with a huge transmission network. This section presents a simple three-bus example as an application and illustration of the general model. In this example, we consider the specific type of transmission investment: building new transmission lines or updating existing ones. Look at the grid in figure 2, where there are 3 interconnected nodes, $n = 1,2,3$. Let $\ell = 1,2,3$ index the lines connecting nodes 2 and 3, nodes 1 and 3 and nodes 1 and 2, respectively. Originally, each line has some capacity. The capacities of line 1 and line 2 are so large that they are never congested. Besides, they have the same impedance. Let k_0 denote the original capacity of line 3. To make things interesting, suppose that k_0 is less than the socially efficient capacity.

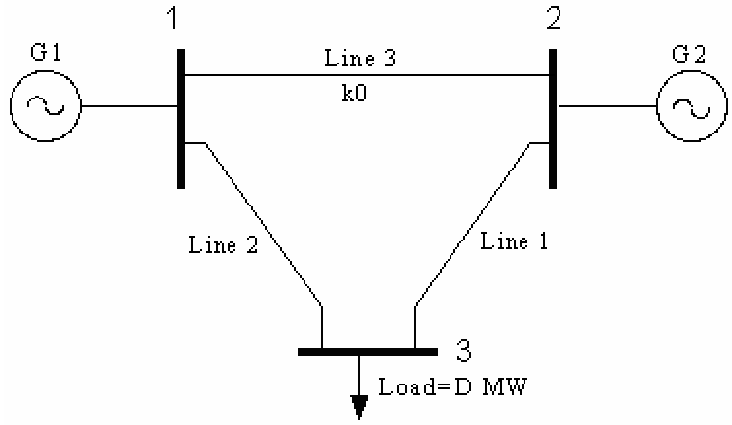

Figure 2: A three-node network

A generator is attached to nodes 1 and 2, respectively, denoted by G_1 and G_2. G_1's cost function is $C_1(q_1)$ and G_2's cost function is $C_2(q_2)$, where q_1 and q_2 are the amount of power generated by G_1 and G_2, respectively, and $C_n(\cdot)$ for $n = 1,2$ are strictly convex functions that satisfy $C_n(0) = 0$. The only load of a constant D MW is located at node 3. There is an investment firm that produces transmission capacity by building or updating transmission circuits. It only chooses to build lines between nodes 1 and 2, since the other two lines already have enough capacities and the price of new transmission will be zero. The investment firm's cost function is $C(I)$, where I is the additional capacity created on the corridor between nodes 1 and 2.

At a feasible dispatch (q_1, q_2), the load must be satisfied, that is $q_1 + q_2 = D$ and the flow along each line should not exceed the line's capacity limit. In this example, since we assume that lines 1 and 2 have large enough capacities, we are only concerned about the flow along line 3. Given the dispatch (q_1, q_2) and newly built capacity *I*, the flow along line 3 from node 1 to node 2 can be written as

$$f_{12} = \alpha(I)(q_1 - q_2)$$

Where $\alpha(I)$ is the distribution factor[10]. Clearly, the flow of power along this line depends on the dispatch, and the capacity enhancement on the line.

A feasible allocation consists of a dispatch (q_1, q_2) and a capacity investment *I*, such that $-(k_0 + I) \leq \alpha(I)(q_1 - q_2) \leq k_0 + I$. Then the optimal allocation $((q_1^*, q_2^*), I^*)$ of this economy can be derived by solving the following problem:

$$\min_{q_1, q_2, I \geq 0} C_1(q_1) + C_2(q_2) + C(I) \tag{24}$$

$$\text{s.t. } q_1 + q_2 = D$$

$$-(k_0 + I) \leq \alpha(I)(q_1 - q_2) \leq k_0 + I$$

Like before, assume that $\alpha(\cdot)$ is such that the set of feasible allocations is convex. Also, assume for simplicity that the above problem has an interior solution and, without loss of generality, that at that solution $\alpha(I^*)(q_1^* - q_2^*) \geq 0$ [11]. Let λ and μ be the Lagrangian multipliers of the above constraints, respectively. Then the first order conditions for an interior solution are:

$$\frac{\partial C_1(q_1^*)}{\partial q_1} = \lambda - \mu\alpha(I^*)$$

$$\frac{\partial C_2(q_2^*)}{\partial q_2} = \lambda + \mu\alpha(I^*)$$

$$\frac{\partial C(I^*)}{\partial I} = \mu - \mu\frac{\partial \alpha(I^*)}{\partial I}(q_1^* - q_2^*) \tag{25}$$

$$q_1^* + q_2^* = D$$

$$\alpha(I^*)(q_1^* - q_2^*) = k_0 + I^*$$

Note that λ is the social cost of satisfying one more MW at node 3 and μ is the marginal social benefit from one MW enhancement in the transmission capacity of line 3. The first two equations state that at the optimal allocation, the private cost of one more MW generated at

[10] Here $\alpha_1^{12} = \alpha_2^{12} = \alpha$, because line 1 and line 2 have the same impedance.

[11] Sufficient conditions for an interior solution would be that marginal costs of generation and investments are 0 when evaluated at 0.

node n for $n = 1,2$ must equal the social cost of one more MW at node n for $n = 1,2$, which is equal to the social cost (λ) of supplying one more MW at node 3 plus the social cost of transmitting it to node n. The third equation dictates that marginal private cost of the transmission investment in line 3 be equal to its marginal social benefit. This social benefit consists of the social benefit of the capacity enhancement on line 3 less the social cost from the change in the flow along that line due to the change in the distribution factor. $\frac{\partial \alpha(I^*)}{\partial I}(q_1^* - q_2^*)$ is the change in the flow on line 3 for a given dispatch, caused by the capacity enhancement and reflects the externality introduced by the transmission investment. The last two equations are the market-clearing conditions for electricity and transmission. It follows that at an interior efficient allocation $((q_1^*, q_2^*), I^*) >> 0$,

$$\frac{\partial C_2(q_2^*)}{\partial q_2} - \frac{\partial C_1(q_1^*)}{\partial q_1} = 2\frac{\frac{\partial C(I^*)}{\partial I}}{1 - \frac{\partial \alpha(I^*)}{\partial I}(q_1^* - q_2^*)}\alpha(I^*) \tag{26}$$

This equation gives the relation between the optimal generations and investment and will be used to for comparison between the optimal allocation and the competitive equilibrium.

Now look at the market outcome. Let t be the transmission price on the newly built line. A competitive equilibrium of this economy consists of an allocation $((q_1^*, q_2^*), I^*)$ and a price vector (p_1, p_2, p_3, t) that satisfy the following conditions:

1. The investment firm maximizes its profits, given t: I^* solves

$$\max_{I \geq 0} tI - C(I) \tag{27}$$

2. Each generator maximizes its profit, given its respective nodal price: q_n^* for $n = 1,2$ solves

$$\max_{q_n \geq 0} p_n q_n - C_n(q_n) \tag{28}$$

3. Markets clear:

$$q_1^* + q_2^* = D$$
$$\alpha(I^*)(q_1^* - q_2^*) = k_0 + I^*$$

4. No arbitrage opportunity exists:

$$p_3 = p_1 + \alpha(I^*)t \quad (29)$$

$$p_3 = p_2 - \alpha(I^*)t \quad (30)$$

Assume that the investment and production levels are strictly positive in equilibrium. Then conditions (27) and (28) can be replaced by the corresponding necessary and sufficient conditions for profit maximization as follows:

$$\frac{\partial C(I^*)}{\partial I} = t \quad (31)$$

$$\frac{\partial C_n(q_n^*)}{\partial q_n} = p_n,\ n = 1,2 \quad (32)$$

Equation (31) says that the investment firm will choose the investment level that equalizes its marginal cost with the transmission price. Likewise, each generator maximizes its profit by having the marginal cost of its generation equal the electricity price at its own node, as indicated by each equation in (32). Substituting (32) into (29) and (30), we get

$$p_3 = \frac{\partial C_1(q_1^*)}{\partial q_1} + \alpha(I^*)t$$

$$p_3 = \frac{\partial C_2(q_2^*)}{\partial q_2} - \alpha(I^*)t$$

These two equations together with (31) yield

$$\frac{\partial C_2(q_2^*)}{\partial q_2} - \frac{\partial C_1(q_1^*)}{\partial q_1} = 2\frac{\partial C(I^*)}{\partial I}\alpha(I^*) \quad (33)$$

Comparing equations (26) and (33), we can see that unless $\frac{\partial C(I)}{\partial I} = 0$, that is, unless investment does not affect the power distribution factor α, we cannot guarantee that the competitive equilibrium is efficient. This divergence results from the effect of transmission investment on the distribution factor and the flow structure of the network. In the social optimality problem, this effect is internalized, while in the market, the investment firm does not take it into account. Therefore, the resulting transmission investment may be inefficient due to the externality introduced by the new investment[12]. This result is consistent with that

[12] Note that the investment in line 3 also changes the distribution factors of line 2 and line 3. But we do not need to worry

derived from the large model in section 3. In spite of the externality, some government intervention can still achieve efficiency via a decentralized market mechanism. For example, given enough information, we can apply an ad valorem tax rate equal to $\frac{\partial \alpha(I^*)}{\partial I}(q_1^* - q_2^*)$. This is shown in Appendix 2.

2.5 Capacitor-induced Capacity Enhancement

In this section, we will give an example of a different type of transmission investment, placing control. A specific control method, adding capacitors is considered here. As a capacitor, after being installed, is switched on only in case a contingency, contingency is much relevant to this type of capacity enhancement. Therefore, we will need to modify the model in section 3 by incorporating the contingency constraints.

Adding capacitors enhance the capacity of some lines under contingency but does not change any capacity under normal conditions. Different from building new lines, this type of transmission investment does not affect the line impedances of the network. Hence, the power distribution factors under normal conditions and contingency will be the same before and after a capacitor is installed. From the results obtained in section 3, we should expect that transmission investment via capacitors in equilibrium will be efficient. This is illustrated in the following example.

Consider the 3-node transmission network illustrated in figure 3. Nodes 2 and 3 are connected by line 1, nodes 1 and 2, by line 3 and buses 1 and 3, by two lines, line 21 and line 22. Suppose that line 1 and line 3 have the same impedance and so do line 21 and line 22. Further, the impedance of line 1 is half of that of line 21, so that each corridor has the same impedance. For simplicity assume that lines 1 and 3 have large enough capacities so that they are never congested, whether in the normal conditions or in a contingency. Each of the two lines that connect nodes 1 and 3, on the other hand, has a capacity of k_1, determined by the voltage or stability limit.

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about the flow constraints of those two lines, because they are never congested.

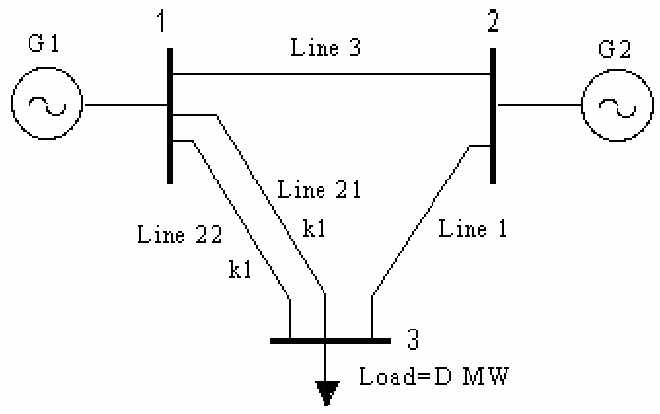

Figure 3: The three-node network under normal conditions

In a contingency, line 21, but not any other line will fail. Suppose that a 20% capacity pre-reserved for line 22 will be released in the contingency, so that the maximum flow allowed through this remaining line when line 21 fails will be $k_2 = 1.2k_1$. Capacities k_1 and k_2 should not be interpreted as a "physical limit" on the flow transmitted through the lines but as "*operational limit*" that results from the satisfaction of the disturbance performance criteria for the network. The network under the contingency is shown in figure 4.

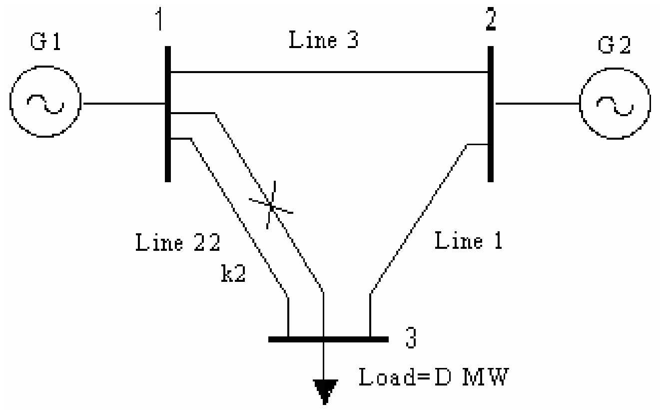

Figure 4: Contingency with no capacitor

As in section 4, a generator is attached to nodes 1 and 2, respectively, denoted by G_1 and G_2. Their cost functions are $C_1(q_1)$ and $C_2(q_2)$, respectively and satisfy the bunch of assumptions given in the preceding section. At node 3, there is a constant load of D MW. There is also an investment firm that can increase the capacity of the network by installing capacitors. As mentioned above, a capacitor is a device that is installed at an appropriate node and that can be switched on in case of a contingency. When the capacitor is switched on, the maximum acceptable flow on a given line will be enhanced by certain units. Specifically for this example, when the capacitor is switched on, the capacity of line 22 will become $k_2 + I$ in the contingency, where I is the capacitor-induced capacity enhancement[13]. The magnitude of I is a decision variable of the investment firm. The cost of increasing the contingent capacity by I units is given by, *C(I)*, where again, $C(\cdot)$ has the same properties as mentioned earlier . Figure 5 illustrates the network under the contingency when a capacitor is installed and switched on.

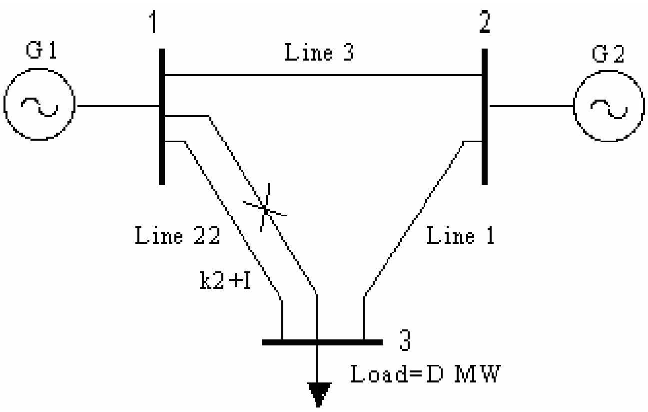

Figure 5: Contingency with capacitor installed and switched on

In order to satisfy the load at node 3, the total generation of the system must satisfy $q_1 + q_2 = D$. However, not every dispatch (q_1, q_2) is allowable. Only those that induce flows on the lines 21 and 22 that respect their capacity constraints are allowed. Given the basic data of the network, in normal circumstances the flow through lines 21 and 22 will be given by $f_{21}(q_1, q_2) = f_{22}(q_1, q_2) = \frac{1}{3}q_1 + \frac{1}{6}q_2$. This flow should not exceed the maximum acceptable flow of k_1. Similarly, if a contingency occurs, and line 22 becomes the only line that remains connecting nodes 1 and 3, the flow through that line will be $f_{22}^{c}(q_1, q_2) = \frac{1}{2}q_1 + \frac{1}{4}q_2$, which should be no greater than $k_2 + I$. The foregoing discussion suggests that a feasible allocation $((q_1, q_2), I)$ must satisfy

[13] Actually, the contingency capacity limits of lines 1 and 3 will also be enhanced by the capacitor. But we do not need to consider these enhancements, because these two lines already have enough capacities, both in normal conditions and in the contingency.

$$q_1 + q_2 = D$$

$$\frac{1}{3}q_1 + \frac{1}{6}q_2 \le k_1$$

$$\frac{1}{2}q_1 + \frac{1}{4}q_2 \le 1.2k_1 + I$$

Among the feasible allocations, those that minimize the social cost are efficient allocations. Formally, an efficient allocation $\left(\left(q_1^*, q_2^*\right), I^*\right)$ solves the following problem:

$$\min_{q_1, q_2, I \ge 0} C_1(q_1) + C_2(q_2) + C(I) \tag{34}$$

$$\text{s.t. } q_1 + q_2 = D$$

$$\frac{1}{3}q_1 + \frac{1}{6}q_2 \le k_1 \tag{35}$$

$$\frac{1}{2}q_1 + \frac{1}{4}q_2 \le 1.2k_1 + I \tag{36}$$

Since the cost functions are assumed to be strictly convex and the constraints are linear, this problem has a unique solution.

Before solving this problem, note that for every dispatch (q_1, q_2), the associated flow through line 22 in normal conditions is lower than the flow in the contingency: $\frac{1}{3}q_1 + \frac{1}{6}q_2 = \frac{2}{3}\left(\frac{1}{2}q_1 + \frac{1}{4}q_2\right)$. As a result, in the absence of a capacitor ($I = 0$) constraint (35) will not bind, or else constraint (36) would be violated. In other words, the contingency capacity limit of line 22 causes the capacities of lines 21 and 22 to be underutilized in normal circumstances. Hence, the benefit of adding a capacitor consists of allowing a more efficient use of a the line capacities under normal conditions and enhancing contingency capacities. Obviously, this benefit should be compared to the cost of the capacitor and the incremental cost of the new dispatch induced.

Now solve problem (34) for the efficient allocation $\left(\left(q_1^*, q_2^*\right), I^*\right)$. Let λ, μ, and η be the Lagrangian multipliers of the constraints in the problem, respectively. Then the FOCs are:

$$\frac{\partial C_1\left(q_1^*\right)}{\partial q_1} \ge \lambda - \frac{1}{3}\mu - \frac{1}{2}\eta \text{ with equality if } q_1^* > 0 \tag{37}$$

$$\frac{\partial C_2(q_2^*)}{\partial q_2} \geq \lambda - \frac{1}{6}\mu - \frac{1}{4}\eta \text{ with equality if } q_2^* > 0 \quad (38)$$

$$\frac{\partial C(I^*)}{\partial I} \geq \eta \text{ with equality if } I^* > 0 \quad (39)$$

$$q_1^* + q_2^* = D \quad (40)$$

$$\frac{1}{3}q_1^* + \frac{1}{6}q_2^* \leq k_1 \text{ with equality if } \mu > 0 \quad (41)$$

$$\frac{1}{2}q_1^* + \frac{1}{4}q_2^* \leq 1.2k_1 + I^* \text{ with equality if } \eta > 0 \quad (42)$$

To understand the above conditions, consider an interior efficient allocation $((q_1^*, q_2^*), I^*) >> 0$. Since the generation at both nodes is positive, constraints (37) and (38) are satisfied with equality. By inspection, this implies that the marginal cost of a MW at node 1 is lower than the marginal cost of a MW at node 2. Hence, if we could generate Δq additional units at the cheaper node 1 and Δq less units at the costly node 2, we would save the amount $\frac{\partial C_2(q_2^*)}{\partial q_2}\Delta q - \frac{\partial C_1(q_1^*)}{\partial q_1}\Delta q$ and still satisfies the load. The problem is that we cannot transfer Δq units of generation from G_2 to G_1 without violating the contingency constraint (42). Therefore, if we want to enjoy the above savings we have to relax the contingency constraint by means of an increase in the operational capacity of line 22 under the contingency. We should increase this operational capacity by a small unit as long as its cost is no bigger than the savings induced by the redispatch that this investment allows. At the optimum, the marginal cost of the capacity should be equal to or higher than its marginal benefit:

$$\frac{\partial C(I^*)}{\partial I} \geq \frac{\partial C_2(q_2^*)}{\partial q_2}\Delta q - \frac{\partial C_1(q_1^*)}{\partial q_1}\Delta q$$

And this is precisely one of the implications of the FOCs (37)-(42). To see this, note that since $I^* > 0$, condition (39) holds with equality, $\frac{\partial C(I^*)}{\partial I} = \eta$. By assumption, the marginal cost of capacitor-induced capacity is positive, so $\eta > 0$ and consequently constraint (42) is binding. Let us consider two cases, one with constraint (41) being unbinding and the other that constraint being binding. If constraint (41) does not bind at the optimal allocation, $\mu = 0$. A unit of additional capacity in case of a contingency allows us to change the injections in nodes 1 and 2, by Δq_1 and Δq_2, respectively, where Δq_1 and Δq_2 satisfy

$$\frac{1}{2}\Delta q_1 + \frac{1}{4}\Delta q_2 = 1$$

$$\Delta q_1 + \Delta q_2 = 0$$

This means that the unit of additional capacity allows us to redispatch in a way that $\Delta q_1 = 4$ and $\Delta q_2 = -4$. That is, G_1 produces 4 more units and G_2 produces 4 less units. The saving in the generation cost induced by this new dispatch is

$$-\frac{\partial C_2(q_2^*)}{\partial q_2}\Delta q_2 - \frac{\partial C_1(q_1^*)}{\partial q_1}\Delta q_1 = -4\left(\left(\lambda - \frac{1}{3}\mu - \frac{1}{2}\eta\right) - \left(\lambda - \frac{1}{6}\mu - \frac{1}{4}\eta\right)\right)$$
$$= \frac{2}{3}\mu + \eta = \eta$$

Now consider the second case in which constraint (41) is binding. Although it saves and the demand is still satisfied if we could have G_1 produce Δq ($\Delta q > 0$) more and G_2 produce Δq less, this transfer would not be feasible even with the additional unit of contingency capacity, because at the new dispatch constraint (41) would be violated:

$$\frac{1}{3}\Delta q - \frac{1}{6}\Delta q > 0, \forall \Delta q > 0$$

In other words, we can only have $\Delta q = 0$ and no cost would be saved. Hence, the marginal benefit of one more unit enhancement in the contingency capacity is zero, which is less than the marginal cost of the capacity $\frac{\partial C(I^*)}{\partial I}$. In all, the marginal benefit of the investment is no greater than its marginal cost at the optimal allocation.

Knowing the efficient dispatch and investment level, we wonder if they can be decentralized by means of a price system and competitive markets. In the following definition of economic equilibrium, there will be nodal prices and two different transmission changes. Both transmission charges are related to congestion on the 1-3 corridor. One charge is associated to the transmission on the corridor under normal conditions and the other, to the transmission under the contingency. The generators and investment firm will take these prices as given and make their generation and investment decision optimally.

An allocation $((q_1^*, q_2^*), I^*)$ and a price vector (p_1, p_2, p_3, t, π) constitute a *competitive equilibrium* of this economy if the following conditions are satisfied:

1. Each generator G_n, for $n = 1,2$, chooses its generation level q_n^* so as to maximize its profit given its own nodal price p_n:

$$p_n q_n^* - C_n(q_n^*) \geq p_n q_n - C_n(q_n), \forall q_n \geq 0, n = 1,2 \tag{43}$$

2. The investment firm chooses the investment level I^* so as to maximize its profit given the contingency transmission charge π:

$$\pi I^* - C(I^*) \geq \pi I - C(I), \forall I \geq 0 \tag{44}$$

3. Power market clears:

$$q_1^* + q_2^* = D \tag{45}$$

4. Transmission market clears:

$$\frac{1}{3}q_1^* + \frac{1}{6}q_2^* \leq k_1 \text{ with equality if } \mu > 0 \tag{46}$$

$$\frac{1}{2}q_1^* + \frac{1}{4}q_2^* \leq 1.2k_1 + I^* \text{ with equality if } \eta > 0 \tag{47}$$

5. No arbitrage opportunity exists:

$$p_3 = p_1 + \frac{1}{3}2t + \frac{1}{2}\pi \tag{48}$$

$$p_3 = p_2 + \frac{1}{6}2t + \frac{1}{4}\pi \tag{49}$$

Note that the capacitor only enhances the contingency capacity. Accordingly, the investment firm will collect the congestion charge in the contingency. Conditions (46) and (47) require that the demand for transmission should not exceed the capacity both under normal conditions and under the contingency. Besides, the associated transmission charge is positive only if the demand for transmission equals the capacity. To understand conditions (48) and (49), note that if we inject one MW at node 1 and eject it at node 3, in normal circumstances $\frac{1}{3}$ of the MW will transit through each of lines 21 and 22. In a contingency,

$\frac{1}{2}$ of the MW will move along the remaining line 22. Therefore, each MW injected at node 1 and withdrawn at node 3 must pay $\frac{1}{3}$ of the price of transmission on line 21 and line 22 together under normal conditions and $\frac{1}{2}$ of the price of transmission on line 22 under the contingency. If we add the power price at node 1, the cost of buying one MW at that node and transmitting it to node 3 is $p_1 + \frac{1}{3}2t + \frac{1}{2}\pi$. Condition (48) states that this cost should equal the price that one would obtain by selling this MW at the destination node 3. A similar interpretation applies to condition (49). The contingency congestion charge is paid not only when a contingency occurs, but also when it does not.

The necessary and sufficient conditions for problems (43) and (44) are:

$$\frac{\partial C_n(q_n^*)}{\partial q_n} \geq p_n \text{ with equality if } q_1^* > 0 \tag{50}$$

$$\frac{\partial C(I^*)}{\partial I} \geq \pi \text{ with equality if } I^* > 0 \tag{51}$$

In all, a competitive equilibrium is characterized by conditions (50) and (51) and conditions (45) through (49).

By comparison, we see that if an allocation $((q_1^*, q_2^*), I^*)$ solves the social optimum problem (34) with associated Lagrangian multipliers (λ, μ, η), then the same allocation together with the price vector (p_1, p_2, p_3, t, π) defined by

$$p_1 = \lambda - \frac{1}{3}\mu - \frac{1}{2}\eta$$

$$p_2 = \lambda - \frac{1}{6}\mu - \frac{1}{4}\eta$$

$$p_3 = \lambda$$

$$t = \frac{\mu}{2}$$

$$\pi = \eta$$

is a competitive equilibrium. Conversely, if an allocation $\left(\left(q_1^*, q_2^*\right), I^*\right)$ and a price vector $\left(p_1, p_2, p_3, t, \pi\right)$ constitute a competitive equilibrium, then the same allocation together with the Lagrangian multipliers $\left(\lambda, \mu, \eta\right)$ defined by

$$\lambda = p_3$$

$$\mu = 2t$$

$$\eta = \pi$$

solves the social optimum problem (34).

The above analysis shows that the competitive equilibrium is efficient. In particular, the competitive equilibrium induces the optimal amount of capacitor-induced capacity enhancement. The reason is that adding capacitors has no effect on the distribution factors under normal conditions or under a contingency. Hence, the flows of power along the lines for any given set of injections is unchanged by the investment. Therefore, as long as it leaves the flows of power for any given set of net injections unaffected, transmission investment will not induce externalities that cause the market to fail even in the presence of loop flows.

The three-node network is miniature of more complicated, meshed networks and the result from this simple example is valid for a network with more nodes. In addition, the model here applies to other control methods such as adding SVC (Static Var Compensator) and STATCOM (Static Synchronous Compensator), because adding those controllers does not change the distribution factors, either. Therefore, the result from this example can be extended to the general control approach.

2.6 Grid Expansion

So far, we have focused on transmission investment within a fixed grid topology. That is, no line is built between a pair of nodes that, originally, are not directly connected by a line before the investment. The main result from the preceding section is that given a grid topology, capacity enhancement that changes the distribution factors introduces an externality that may cause the investment in equilibrium to be inefficient. For referral convenience, let us call this *type I transmission investment*. There is no market failure with capacity enhancement that does not affect the distribution factors and the investment in equilibrium is efficient or optimal. Let us call this *type II transmission investment*. We can say that type II transmission investment is constrained optimal, in the sense that it is optimal given the grid topology. In this section, we will consider the case of grid expansion, in which the grid topology itself it also to be determined. And the efficient or optimal allocation in this case is, in comparison, fully optimal or socially optimal. Apparently, type I transmission investment is usually not fully optimal, since it is not even constrained optimal. So the issue of full optimality versus constrained optimality is only relevant to type II transmission investment. Then is type II transmission investment optimal when the grid topology is endogenous? The answer is no.

Consider a fixed set of N nodes. Given a grid topology with the set of existing corridors denoted by L_1, the equilibrium allocation CE_1 in the case of type II transmission investment is efficient and yields the highest social surplus SS_1^* that can be achieved under this very topology. Given a different grid topology with the set of existing corridors denoted by L_2, the corresponding equilibrium allocation CE_2 yields the highest social surplus SS_2^* under this different topology. Generally, $SS_1^* \neq SS_2^*$. If $SS_1^* > SS_2^*$, then CE_2 is not socially optimal (although constrained optimal), since there exists at least one other allocation CE_1 that results in a higher social surplus than does CE_2. Whether CE_1 is socially optimal depends on whether the social surplus resulting from it is the highest among all the constrained optimal allocations associated with all the possible topologies. If it is, CE_1 will be socially optimal. In general, there can be many different grid topologies for a fixed set of nodes and each of them has an equilibrium which is optimal under that specific topology. Among them, some grid topology and the corresponding competitive equilibrium with type II transmission investment amounts to the highest social surplus. That very grid topology and the transmission investment in the corresponding equilibrium are fully optimal, while the others are only constrained optimal, and not fully optimal. What follows is a numerical example, illustrating the difference between constrained and full optimality as well as our reasoning above.

Figure 6 gives three nodes, 1, 2, and 3 and originally there is no transmission line between any two of them. Generators G_1 and G_2 are located at node 1 and node 2 and have the cost function $C_1(q) = 20q + q^2$ and $C_2(q) = 10q + \frac{1}{2}q^2$, respectively. The only load of 1000MW is located at node 3 and there is no generator at that node. Given the three nodes, there can be at most three corridors, one between each pair of nodes (denoted in dashed lines). There is an investment firm that makes type II transmission investment according to the cost function $C(I_1, I_2, I_3) = I_1^2 + I_2^2 + I_3^2 + \frac{1}{3}(I_1 + I_2 + I_3) - 1{,}000{,}000$, where I_n, for $n = 1,2,3$ is the new capacity built on corridor *n*. We will ignore contingency constraints in this example. Including them can only affect the numerical results and complicate the calculation, but will not change anything essential.

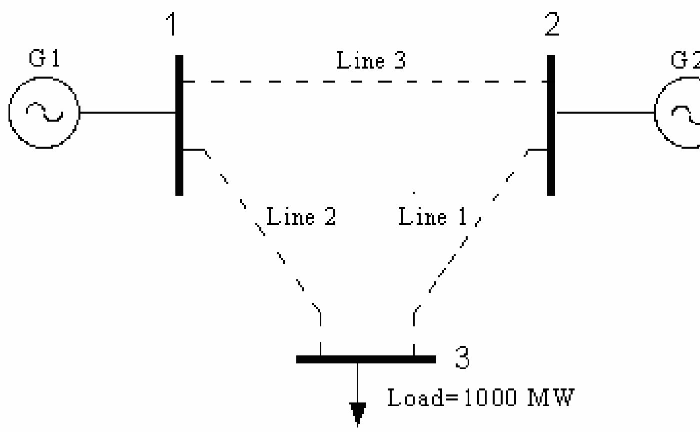

Figure 6: Three nodes

Let us first consider the case when there are two lines, 1 and 2 in the beginning, with one between nodes 1 and 3 and the other between nodes 2 and 3. For calculation simplicity, assume that the impedance is the same for the two lines and their original capacities are both normalized to zero. The investment firm makes investment within this grid topology. A competitive equilibrium of this economy consists of a price vector $(p_1, p_2, p_3, t_1, t_2)$, production plans of the generators (q_1^*, q_2^*), and an investment plan of the investment firm (I_1^*, I_2^*)[14]. The concrete numbers are shown in Figure 7. As shown earlier, the equilibrium allocation is the same as the allocation derived from solving the optimality problem. That is, this competitive equilibrium is optimal given the two-line topology.

[14] Here we have $I_3^* = 0$, since there is no investment between node 1 and node 2.

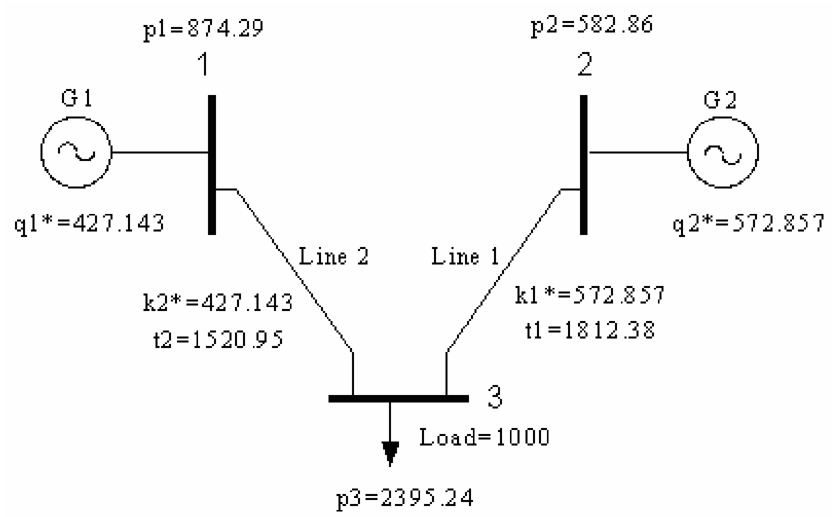

Figure 7: Two-line equilibrium

There is another equilibrium in which three lines are built. Figure 8 illustrates this equilibrium: a price vector $(p_1, p_2, p_3, t_1, t_2, t_3)$, production plans (q_1^*, q_2^*) and an investment plan (I_1^*, I_2^*, I_3^*). Similarly, we can show that this equilibrium is optimal given the three-line topology.

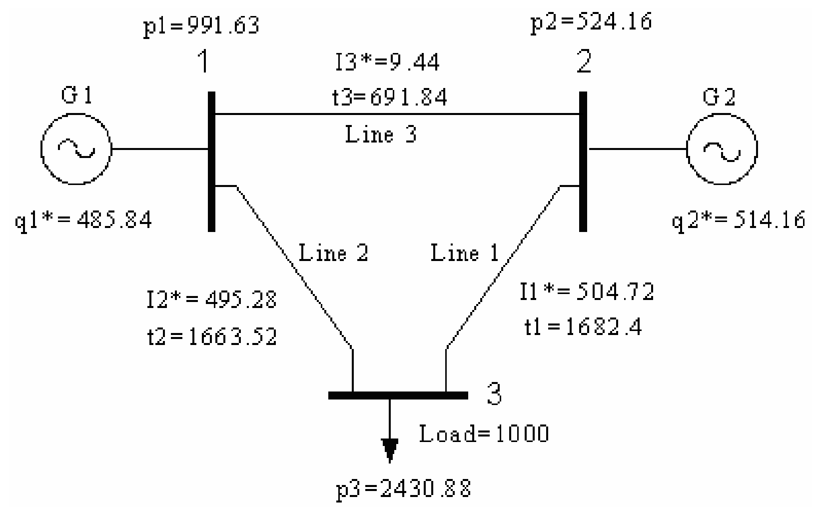

Figure 8: Three-line equilibrium

So the type II transmission investment in both the two-line and three-line equilibria above is constrained optimal. The social cost in the equilibrium with two lines is $204,755, which is lower than $222,869, the cost in the equilibrium with three lines. This is reasonable, since in the two-line equilibrium more electricity is produced by the cheaper generator, G_2.

Hence, the three-line equilibrium is not socially optimal. To see if the two-line equilibrium is socially optimal, we need to compare it with all the other equilibria. As we can not tell which equilibrium the economy will end up with, it is guaranteed that the equilibrium will be socially optimal, although it must be constrained optimal.

From this section and the preceding one, we find that transmission investment that does not change the flow structure of the network may not be socially optimal, but is always constrained optimal given the grid topology. This result is consistent with our earlier finding that transmission investment introduces an externality if its affects the flow of power along the lines for any given set of injections. In the case of grid expansion, both type I and type II investments change the distribution factors, since the line topology is affected. Therefore, there is a market failure whether type I or type II investment is made. Here we do not use the derivatives as we do in section 5, because building a line between two nodes that are not originally connected causes a jump in the capacity of that line, so differentiation no longer applies. However, the cause for the market failure in grid expansion is still the externality created by loop flows.

So far, we have been playing with the model of certainty. In reality, however, there can be many uncertain aspects in power and transmission markets. For example, instead of being deterministic, the loads or generation technology may vary across different states. To consider these facts, we introduce uncertainty to the previous model by allowing for state-dependent preferences and technology. See Appendix 3.

2.7 Concluding Remarks

Market-based transmission investment under perfect competition is not efficient, due to the externalities caused by loop flows. However, why and how loop flows create externalities is a question that has never been answered. The externalities are associated with loop flows, but it is not that externalities are introduced whenever there are loop flows. This paper clarifies the nature of loop externalities and points out when loop flows are a problem and when they are not. In doing so, we develop a model with endogenized transmission investment. From the model, we conclude that transmission investment introduces an externality if and only if it affects the flow of power along the lines for a given set of injections. Interestingly these externalities are not like those in the usual sense, in that neither the consumers' preferences nor the generators' technologies are directly affected by the transmission investment actions. Instead, the externalities come in because transmission investment changes the physical aspects of the network, i.e. the way in which electricity is transmitted. To illustrate the general model, we analyze two different types of transmission investment in two three-node examples, respectively. The first type of investment is building new transmission links and the second is introducing additional control capability. We find that in a meshed network, transmission investment in lines at a competitive equilibrium is not guaranteed to be optimal, while investment in control is. This result is consistent with our earlier conclusion. Building new lines results in externalities, because it affects the flow of power along the lines for a given set of injections. In contrast, placing control leaves the flow structure unchanged, so there is no externality even in the presence of loop flows. These results apply to the case of grid expansion and also hold in an economy with uncertainty.

Now we can answer the questions raised in the beginning of the paper. The addition or removal of lines are not necessary for markets to fail: the competitive equilibrium will not be efficient even if the grid topology is restricted to remain the same but upgrades of line capacities that change the power flow are allowed. The change in the set of feasible injections itself is not responsible for the market failure: as long as the line capacities are changed in a flow-preserving way, there will be no externalities associated with the investment. Finally, the fact that injections in one bus affect the set of allowable injections in other buses is not the source of externalities, either. The truth of the last two statements can be seen by observing a two-bus network: in such a network, the flow structure is always the same; namely, each MW injected in one bus transits the only line, independently of its capacity. In conclusion, the externalities leading to inefficient transmission investment are caused by loop flows, but the existence of loop flows does not necessarily result in such externalities. It does, only when the transmission investment changes the flows of power along the lines for any given of injections.

Our paper points out an externality that is only found in the power market. There is no objection that regulation is needed in transmission expansion planning. However, so far, most emphasis has been focused on the regulation necessary to control market power or natural monopoly. Admittedly, these attributes can lead to inefficient transmission investment, but this is not a result specific to the power industry. Loop externalities, instead are something that is not available in other industries. They are inherent in power markets regardless of the market structure, but have never been given attention. Our paper warns that even if the power and transmission investment markets were competitive and each market participant were forced to price as if it were a price taker, market failure could still occur due to the loop externalities we identify. Admittedly, things may be more complicated in investment markets with market power, because in that case not only the loop externalities but also market power will contribute to inefficient transmission investment. But regulation that only mitigates market power is not enough to achieve efficient transmission investment. Clearly the loop flow externalities must also be taken into account in defining property rights and creating incentives for efficient transmission investment and in devising regulation schemes to achieve second-best results.

The model in this paper is static and everything takes place one-shot. The investment firms make transmission investment and have the costs recovered immediately and once. In reality, a transmission investment project usually takes a relatively long time to be completed; the built facilities are used for many periods afterwards and the investment cost is recovered period by period, instead of all at once; there may be entry or exit of market participants in the long run; and the grid may evolve gradually with transmission investment. Although these attributes are not reflected in our model here, it does address the externalities created by loop flows that also exist in a dynamic scenario. In future we can develop a dynamic model, but the inclusion of the dynamic attributes should not change the result that transmission investment introduces an externality if it affects the flow structure of the network. The externalities associated with loop flows will always be there, whether we do static or dynamic modeling. A meaningful and important extension of our current work will be to see how efficient transmission investment can be achieved through proper market design that addresses those externalities.

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2.9 Appendices

Appendix 1: Derivation of equation (1)

The voltage at bus *i* is a sinusoid waveform whose instantaneous value at time *t* is

$$v_i(t) = V_i \sin(\omega t + \theta_i)$$

where V_i is the magnitude (amplitude) of the sinusoid waveform, $\omega = 2\pi \times 60$ is the frequency of the waveform in radians per second and θ_i is its phase angle. Given the impedances Z_{ij}, the real power flow over the line from *i* to *j* is given by

$$f_{i\to j} = \frac{V_i V_j \sin(\theta_i - \theta_j)}{Z_{ij}} \qquad (72)$$

measured in MW. x_i, the net power injection at bus *i* is the sum of the flows $f_{i\to j}$ over all $j = 1,..., N$:

$$x_i = \sum_{j=1}^{N} f_{i\to j} = \sum_{j=1}^{N} \frac{V_i V_j \sin(\theta_i - \theta_j)}{Z_{ij}}, \forall i = 1,..., N \qquad (73)$$

Assume that the voltage magnitude V_i at bus *i* is constant. Without loss of generality, we can set $V_i \equiv 1$ for all $i = 1,..., N$. Then (72) and (73) become:

$$f_{i\to j} = \frac{\sin(\theta_i - \theta_j)}{Z_{ij}}, \forall i, j = 1,\ldots, N \quad (74)$$

$$x_i = \sum_{j=1}^{N} \frac{\sin(\theta_i - \theta_j)}{Z_{ij}}, \forall i = 1,\ldots, N \quad (75)$$

Equations in (75) are the "real power flow equations". Only *N*-1 of these equations are independent, since $\sum_{i=1}^{N} x_i \equiv 0$ in a lossless system. Besides, what matters are the phase angle difference, rather than the angles themselves. Thus, we can set $\theta_N \equiv 0$ and eliminate the *N*th equation in (75). Node *N* can then be regarded as the reference node in the network[15]. Now we have *N*-1 equations with *N*-1 unknowns, $\theta_1,\ldots,\theta_{N-1}$. Given the net power injections $x_1,\ldots,x_{N-1}$, we can solve for the phase angles $\theta_i^* = \theta_i(x_1,\ldots,x_{N-1})$, $i = 1,\ldots, N-1$ and obtain the flow on each line by (74).

Now we make one more simplifying assumption: the phase angle differences, $|\theta_i - \theta_j|$, are very small. Then the following approximation holds: $\sin(\theta_i - \theta_j) \approx \theta_i - \theta_j$. As a result, equations (74) and (75) all become linear, and the solutions $\theta_1^*,\ldots,\theta_{N-1}^*$ are linear in the net injections. That is,

$$\begin{aligned} f_{i\to j} &= \frac{\theta_i^* - \theta_j^*}{Z_{ij}} \\ &= \frac{\theta_i(x_1,\ldots,x_{N-1}) - \theta_j(x_1,\ldots,x_{N-1})}{Z_{ij}} \\ &= \sum_{n=1}^{N-1} \alpha_n^{ij} x_n, \forall i, j = 1,\ldots, N \end{aligned}$$

where α_n^{ij} are functions of Z_{ij}. Since Z_{ij} are constant for a fixed grid, so are α_n^{ij}.

[15] The reference node refers to the node with the phase angle 0. In determining the distribution factors or calculating flows, the reference node can be treated as if it were the only withdrawal node and all the others were injection nodes in the network.

Appendix 2: Proof of the ad valorem tax rate of $\frac{\partial \alpha(I^*)}{\partial I}(q_1^* - q_2^*)$ being the second best

Suppose that the government imposes an ad valorem tax τ on the capacity enhancement of corridor 3. Then the investment firm's profit maximization problem becomes

$$\max_{I \geq 0} t(1-\tau)I - C(I)$$

which has the FOC

$$\frac{\partial C(I^*)}{\partial I} = \tau(1-t)$$

And the other conditions that a competitive equilibrium should satisfy are

$$\frac{\partial C_n(q_n^*)}{\partial q_n} = p_n, \; n = 1,2$$

$$q_1^* + q_2^* = D$$

$$\alpha(I^*)(q_1^* - q_2^*) = k_0 + I^*$$

$$p_3 = p_1 + \alpha(I^*)t$$

$$p_3 = p_2 - \alpha(I^*)t$$

By comparison, we can see that if an allocation $((q_1^*, q_2^*), I^*)$ solves the social optimum problem (24) with associated Lagrangian multipliers (λ, μ), then the same allocation $((q_1^*, q_2^*), I^*)$ together with a price vector (p_1, p_2, p_3, t) and an ad valorem tax rate τ defined by

$$p_1 = \lambda - \alpha(k^*)\mu$$

$$p_2 = \lambda + \alpha(k^*)\mu$$

$$p_3 = \lambda$$

$$t = \mu$$

$$\tau = \frac{\partial \alpha(I^*)}{\partial I}(q_1^* - q_2^*)$$

is a competitive equilibrium. Conversely, if an allocation $((q_1^*, q_2^*), I^*)$ together with a price vector (p_1, p_2, p_3, t) and an ad valorem tax rate $\tau = \frac{\partial \alpha(I^*)}{\partial I}(q_1^* - q_2^*)$ constitute a competitive equilibrium, then the same allocation $((q_1^*, q_2^*), I^*)$ together with the Lagrangian multipliers (λ, μ) defined by

$$\lambda = p_3$$
$$\mu = t$$

solve the social optimum problem (24). So, we can achieve efficiency by imposing the ad valorem tax $\tau = \frac{\partial \alpha(I^*)}{\partial I}(q_1^* - q_2^*)$ on the transmission investment.

Appendix 3: Transmission investment under uncertainty

(1) Model specification and assumptions

There are two commodities in the economy, namely power and the numeraire. The settings and nomenclature about the grid are the same as before. Still assume that only one consumer and one generator are attached to each node. All the existing lines of the network are owned by a single TO. Different from before, there are two periods, $t = 0,1$ and two states of the world, $s = PH, OH$. We may think of states *PH* and *OH* as the peak hour and off-peak hour, respectively. Let π_{PH} and π_{OH} be the probability that state *PH* and state *OH* occurs, respectively, such that $\pi_{PH} + \pi_{OH} = 1$. The true state is revealed in period 1.

Consumers have different endowments and utility functions in different states. Specifically, in state s, consumer n is endowed with a fixed amount of the numeraire ω_n^s and a quasi-linear utility function $u_n^s : R \times R_+ \to R$ such that

$$u_n^s(m_n^s + c_n^s) = m_n^s + \phi_n^s(c_n^s), \forall n = 1,..., N, s = PH, OH$$

where $c_n^s \geq 0$, and $m_n^s \in R$ denote the consumer's consumption of power and the numeraire, respectively in state *s*. $\phi_n^s(c_n^s)$ is assumed to be bounded above with $\frac{\partial \phi_n^s(c_n^s)}{\partial c_n^s} > 0$,

$\frac{\partial^2 \phi_n^s(c_n^s)}{\partial (c_n^s)^2} < 0$, and $\phi_n^s(0) = 0$. So the total amount of the numeraire available in the economy in state *s* is

$$\varpi_s = \sum_{n=1}^{N} \omega_n^s, s = PH, OH$$

Generators' technologies also depend on the state. In state *s*, generator *n* has a production set given by

$$Y_n^s = \{(-z_n^s, q_n^s) : q_n^s \geq 0 \text{ and } z_n^s \geq C_n^s(q_n^s)\}, n = 1,\ldots,N, s = PH, OH$$

where $C_n^s(q_n^s)$ is the generator's cost function in state s, measured in the numeraire in the same state. Assume that $C_n^s(q_n^s)$ is twice differentiable with $\frac{\partial C_n^s(q_n^s)}{\partial q_n^s} > 0$, $\frac{\partial^2 C_n^s(q_n^s)}{\partial (q_n^s)^2} > 0$, and $C_n^s(0) = 0$.

There is a set E of investment firms indexed by $r \in E$. They make investment decisions in period 0, before the true state is revealed. Therefore, no matter which state occurs in period 1, investment firms have to produce with the same technology given in (3) in section 3 and the investment levels are independent of the state. Still let I_ℓ^r be the new transmission capacity on corridor ℓ built by investment firm r, then the total investment on that corridor is $I_\ell = \sum_{r \in E} I_\ell^r$. As the investment affects the distribution factors, they can be expressed as $\alpha_n^\ell((I_v)_{v \in L})$.

(2) Efficient allocation

Like before, an allocation consists of each consumer's consumption plan, each generator's production plan and each investment firm's investment plan. The difference is that under uncertainty, the consumption and production plans depend on the true state of world. The investment plan, which is carried out before the true state is revealed, is state independent. Now an efficient allocation in this economy with uncertainty can defined as follows:

Definition 4: A feasible allocation $\left(\left(\left(m_n^s, c_n^s\right)_{n=1}^N, \left(-z_n^s, q_n^s\right)_{n=1}^N\right)_{s=PH,OH}, \left(-z^r, \left(I_\ell^r\right)_{\ell\in L}\right)_{r\in E}\right)$ in this economy is a specification of a consumption bundle $\left(m_n^s, c_n^s\right)\in R\times R_+$ for each consumer $n=1,\ldots,N$ in each state $s=PH,OH$, a production plan $\left(-z_n^s, q_n^s\right)\in Y_n^s$ for each generator $n=1,\ldots,N$ in each state $s=PH,OH$, and an investment plan $\left(-z^r,\left(I_\ell^r\right)\right)\in Y^r$ for each investment firm $r\in E$, such that

$$\sum_{n=1}^N m_n^s + \sum_{n=1}^N z_n^s + \sum_{r\in E} z^r = \varpi_s \tag{52}$$

$$\sum_{n=1}^N c_n^s = \sum_{n=1}^N q_n^s \tag{53}$$

$$\sum_{n=1}^{N-1} \alpha_n^\ell\left(\left(I_v\right)_{v\in L}\right)\left(q_n^s - c_n^s\right)\le k_\ell^0 + I_\ell,\ \forall \ell\in L \tag{54}$$

for $s=PH,OH$.

Condition (52) requires the equality between the total amount of the numeraire for consumption, production and investment and the amount of the numeraire available in the economy in each state. Condition (53) dictates that the total electricity generation should satisfy the total demand in each state. Condition (54) requires that in each state the flow along every line must not exceed its capacity limit. Comparing these conditions with conditions (5)-(7), we can see that the conditions under uncertainty are nothing more than the conditions under certainty applied to different states, since now everything except the transmission investment is state-contingent.

Among the feasible allocations, those that cannot be improved upon are efficient allocations, defined in the following:

Definition 5: A feasible allocation $\left(\left(\left(m_n^{s*}, c_n^{s*}\right)_{n=1}^N, \left(-z_n^{s*}, q_n^{s*}\right)_{n=1}^N\right)_{s=PH,OH}, \left(-z^{r*}, \left(I_\ell^{r*}\right)_{\ell\in L}\right)_{r\in E}\right)$ is efficient (or optimal) if there is no alternative feasible allocation $\left(\left(\left(m_n^s, c_n^s\right)_{n=1}^N, \left(-z_n^s, q_n^s\right)_{n=1}^N\right)_{s=PH,OH}, \left(-z^r, \left(I_\ell^r\right)_{\ell\in L}\right)_{r\in E}\right)$ such that $\sum_{s=PH,OH}\pi_s u_n^s\left(m_n^s, c_n^s\right)\ge \sum_{s=PH,OH}\pi_s u_n^s\left(m_n^{s*}, c_n^{s*}\right), \forall n=1,\ldots,N$ with strict inequality for at least one agent n.

According to the definition, no other feasible allocation can make an agent better-off than the efficient allocation without hurting the other agents. Under uncertainty, the agent's welfare is measured by the expected utility. Formally, an efficient allocation $\left(\left(\left(m_n^{s*}, c_n^{s*}\right)_{n=1}^{N}, \left(-z_n^{s*}, q_n^{s*}\right)_{n=1}^{N}\right)_{s=PH,OH}, \left(-z^{r*}, \left(I_\ell^{r*}\right)_{\ell\in L}\right)_{r\in E}\right)$ solves the following problem:

$$\max_{\substack{c_n^s, q_n^s, c_\ell^r \ge 0 \\ \forall n=1,\dots,N, \\ \ell\in L, r\in E \\ s=PH,OH}} \sum_{s=PH,OH} \pi_s \left(\sum_{n=1}^{N} \phi_n^s\left(c_n^s\right) - \sum_{n=1}^{N} C_n^s\left(q_n^s\right) \right) - \sum_{r\in E} C^r\left(I_1^r, \dots, I_L^r\right) \tag{55}$$

$$\text{s.t. } \sum_{n=1}^{N} c_n^s = \sum_{n=1}^{N} q_n^s, \; s = PH, OH$$

$$\sum_{n=1}^{N-1} \alpha_n^\ell\left(\left(I_v\right)_{v\in L}\right)\left(q_n^s - c_n^s\right) \le k_\ell^0 + I_\ell, \; \forall \ell \in L, \; s = PH, OH$$

Assume that $\alpha_n^\ell(\cdot)$ are convex to guarantee convexity of the set of feasible allocations. Let $\left(\lambda_s, \left(\mu_\ell^s\right)_{\ell\in L}\right)_{s=PH,OH}$ be the Lagragian multipliers of the constraints above, respectively. Then the FOCs for an efficient allocation are

$$\pi_s \frac{\partial \phi_n^s}{\partial c_n^s}\left(c_n^{s*}\right) \le \lambda_s - \sum_{\ell\in L} \mu_\ell^s \mu_n^\ell\left(\left(I_v^*\right)_{v\in L}\right), \text{ with equality if } c_n^{s*} > 0, \forall n = 1,\dots, N-1$$

$$\pi_s \frac{\partial \phi_N^s}{\partial c_N^s}\left(c_N^{s*}\right) \le \lambda_s, \text{ with equality if } c_N^{s*} > 0$$

$$\pi_s \frac{\partial C_n^s}{\partial q_n^s}\left(q_n^{s*}\right) \le \lambda_s - \sum_{\ell\in L} \mu_\ell^s \mu_n^\ell\left(\left(I_v^*\right)_{v\in L}\right), \text{ with equality if } q_n^{s*} > 0, \forall n = 1,\dots, N-1 \tag{56}$$

$$\pi_s \frac{\partial C_N^s}{\partial q_N^s}\left(q_N^{s*}\right) \le \lambda_s, \text{ with equality if } q_N^{s*} > 0$$

$$\sum_{s=PH,OH} \mu_\ell^s - B \le \frac{\partial C^r}{\partial I_\ell^r}\left(I_1^{r*}, \dots, I_L^{r*}\right), \text{ with equality if } I_\ell^{r*} > 0, \forall \ell \in L, r \in E \tag{57}$$

$$\sum_{n=1}^{N} c_n^{s*} = \sum_{n=1}^{N} q_n^{s*}$$

$$\sum_{n=1}^{N-1} \alpha_n^\ell\left(\left(I_v^*\right)_{v\in L}\right)\left(q_n^{s*} - c_n^{s*}\right) \le k_\ell^0 + I_\ell^*, \text{ with equality if } \mu_\ell^s > 0, \forall \ell \in L$$

for $s = PH, OH$, and where $B = \sum_{s=PH,OH} \sum_{z=1}^{L} \mu_z^s \sum_{n=1}^{N-1} \frac{\partial \alpha_n^z \left(\left(I_v^* \right)_{v \in L} \right)}{\partial I_\ell} \left(q_n^{s*} - c_n^{s*} \right)$. Essentially, the FOCs here are analogous to the FOCs for the social optimum problem under certainty (8), applied to each of the two states.

The last two equations above are the feasibility conditions: aggregate power generation should be equal to the aggregate power consumption, and the line flows induced by the dispatch should satisfy the corresponding capacity constraints. The multipliers μ_ℓ^s, for $\ell \in L$ and $s = PH, OH$ are the marginal social benefit in state *s* from one more MW increase in the capacity of line ℓ, or alternatively, the marginal social cost in state *s* of reducing the capacity on line ℓ by one MW. This social benefit can be positive only when the flow constraint is binding in that state at the efficient allocation. The multiplier λ_s is the marginal social benefit in state *s* of one more MW consumed at the reference node *N*, or equivalently the marginal social cost in state *s* of one more MW produced at node *N*.

The first two sets of conditions necessitate that in each state, for $c_n^{s*} > 0$, the private benefit from one more unit of power supplied at node n be equal to the social cost of supplying it at that node. This social cost, $\lambda_s - \sum_{\ell \in L} \mu_\ell^s \mu_n^\ell \left(\left(I_v^* \right)_{v \in L} \right)$ is equal to the social cost of supplying an additional MW at the residual node, λ_s plus the social cost of transmitting the one MW to node *n*, $-\sum_{\ell \in L} \mu_\ell^s \mu_n^\ell \left(\left(I_v^* \right)_{v \in L} \right)$.

The second two sets of equations state that the private cost of generation of one MW at each node n in state s should equal the social cost of generating an additional MW at that node in that state, unless $q_n^{s*} = 0$, in which case the private cost can be greater than the social cost, which is $\lambda_s - \sum_{\ell \in L} \mu_\ell^s \mu_n^\ell \left(\left(I_v^* \right)_{v \in L} \right)$.

Lastly, equations in (57) require that the marginal private cost of the investment on the capacity of line ℓ be equal to the social benefit of that investment. The social benefit consists of two parts. One is the social benefit of the increased capacity of the line, $\sum_{s=PH,OH} \mu_\ell^s$, which is equal to the sum of the social benefit in each state. The second part B is the social cost for both states incurred by the change in the distribution factors due to the investment on line ℓ.

(3) Arrow-Debreu equilibrium

In this part, we will see what the decentralized outcome of this economy will be. Before defining the market equilibrium, let us introduce the concept of (state) contingent commodities. For every physical commodity $g = 1,...,G$ and state $s = 1,...,S$, a unit of (state) contingency commodity g_s is a title to receive a unit of the good g iff state s occurs. So the number of contingent commodities is $G \times S$, the number of physical commodities times the number of states. In our model, we postulate the existence of a competitive market for each contingent commodity, namely power at each node, transmission on each line and the numeraire, in each state. These markets open before the resolution of uncertainty, that is, at $t=0$ and are for delivery of goods at $t=1$ (they are commonly called forward markets). For each state $s = PH, OH$, there will be electricity prices p_n^s associated with each node, transmission price t_ℓ^s on each corridor and the numeraire price w_s. In the model without uncertainty, the price of the numeraire is normalized to unity. Here, with uncertainty, the price of the numeraire is contingent on the state and not necessarily the same in both states. So, we can not simply normalize the numeraire price in each state to unity. In this economy, there is ex ante trade only, and no ex post trade. What is being purchased (or sold) in the market for a contingent commodity is commitments to receive (or to deliver) amounts of that physical good, if, and when, state s occurs. In period $t = 1$, a state s is revealed, contracts are executed, and every consumer receives a consumption bundle. Observe that although deliveries are contingent, the payments are not. Notice also that for the markets to be well defined it is indispensable that all economic agents be able to recognize the occurrence of state s. That is, information should be symmetric across economic agents. So we assume that the probability π_s is common knowledge.

In each state, each generator decides how much electricity to produce in response to its own nodal price and each consumer decides how much electricity to consume in response to her own nodal price. Given all the nodal prices and transmission prices in both states, the investment firms, before knowing the true state, choose how much extra capacity to build through transmission investment. Like before, the generators, investment firms and TO are owned by the consumers. Then the market equilibrium of this economy with uncertainty can be defined as follows.

Definition 6: An allocation $\left(\left(\left(m_n^{s*}, c_n^{s*}\right)_{n=1}^{N}, \left(-z_n^{s*}, q_n^{s*}\right)_{n=1}^{N}\right)_{s=PH,OH}, \left(-z^{r*}, \left(I_\ell^{r*}\right)_{\ell\in L}\right)_{r\in E}\right)$ and a system of prices for the contingent commodities $(w, p, t) = \left(w_s, \left(p_n^s\right)_{n=1}^{N}, \left(t_\ell^s\right)_{\ell\in L}\right)_{s=PH,OH}$ constitute an Arrow-Debreu equilibrium if:

1. Generators' profit maximization: For each generator $n = 1,\ldots,N$, $\left(-z_n^{s*}, q_n^{s*}\right) \in Y_n^s$ for $s = PH, OH$ satisfies

$$\sum_{s=PH,OH} \left(p_n^s q_n^{s*} - w_s z_n^{s*}\right) \geq \sum_{s=PH,OH} \left(p_n^s q_n^s - w_s z_n^s\right), \forall\left(-z_n^s, z_n^s\right) \in Y_n^s$$

2. Investment firms' profit maximization: For each investment firm $r \in E$, $\left(-z^{r*}, \left(I_\ell^{r*}\right)_{\ell\in L}\right) \in Y^r$

$$\sum_{s=PH,OH} \left(\sum_{\ell\in L} t_\ell^s I_\ell^{r*} - w_s z^{r*}\right) \geq \sum_{s=PH,OH} \left(\sum_{\ell\in L} t_\ell^s I_\ell^r - w_s z^r\right), \forall\left(-z^r, \left(I_\ell^r\right)_{\ell\in L}\right) \in Y^r$$

3. Utility maximization: For each consumer $n = 1,\ldots,N$, $\left(m_n^{s*}, c_n^{s*}\right)_{s=PH,OH}$ solves the problem

$$\max_{\substack{\left(m_n^s, c_n^s\right) \in R\times R_+, \\ \forall s=PH,OH}} \sum_{s=PH,OH} \pi_s \left(m_n^s + \phi_n^s\left(c_n^s\right)\right)$$

$$\text{s.t.} \quad \sum_{s=PH,OH} \left(w_s m_n^s + p_n^s c_n^s\right) \leq \sum_{s=PH,OH} \left(\begin{array}{l} w_s m_n^s + \sum_{j=1}^{N} \theta_n^j \left(p_j^s q_j^{s*} - w_s z_j^{s*}\right) + \\ \sum_{r\in E} \theta_n^r \left(\sum_{\ell\in L} t_\ell^s I_\ell^{r*} - w_s z^{r*}\right) + \theta_n^T \sum_{\ell\in L} t_\ell^s k_\ell^0 \end{array}\right)$$

4. No arbitrage: for any dispatch $\left(x_1^s, \ldots, x_N^s\right)_{s=PH,OH}$ such that $\sum_{n=1}^{N} x_n^s = 0$ for $s = PH, OH$

$$\sum_{s=PH,OH} \left(\sum_{n=1}^{N} p_n^s \left(q_n^s - c_n^s\right) + \sum_{\ell\in L} \sum_{n=1}^{N-1} \alpha_n^\ell \left(\left(I_v^*\right)_{v\in L}\right)\left(q_n^s - c_n^s\right)\right) \geq 0 \qquad (58)$$

5. Market clearing: For each state $s = PH, OH$

$$\sum_{n=1}^{N} m_n^{s*} + \sum_{n=1}^{N} z_n^{s*} + \sum_{r\in E} z^{r*} = \varpi_s \qquad (59)$$

$$\sum_{n=1}^{N} c_n^{s*} = \sum_{n=1}^{N} q_n^{s*}$$

$$\sum_{n=1}^{N-1} \alpha_n^{\ell}\left(\left(I_v^*\right)_{v\in L}\right)\left(q_n^{s*} - c_n^{s*}\right) \le k_\ell^0 + I_\ell^*, \text{ with equality if } t_\ell^s > 0, \forall \ell \in L$$

Note that at any production or investment plan, the profit of a generator or of an investment firm is a nonrandom amount of dollars, as in conditions 1 and 2 above. This is because in the Arrow-Debreu framework, although productions and deliveries of goods depend on the state of the world, the firm is active in all the contingent markets and manages to insure completely.

Now let us characterize the five conditions in the definition above. Condition 1 says that each generator maximizes its total profit across the two states, given its own technology and nodal price: formally, it chooses q_n^{s*}, for $s = PH, OH$ to solve

$$\max_{\substack{q_n^s \ge 0 \\ \forall s = PH, OH}} \sum_{s=PH,OH} \left(p_n^s q_n^s - C_n^s\left(q_n^s\right)\right)$$

The necessary and sufficient conditions for q_n^{s*} to solve the problem are

$$p_n^s \le w_s \frac{\partial C_n^s}{\partial q_n^s}\left(q_n^{s*}\right), \text{ with equality if } q_n^{s*} > 0, \forall n = 1,\dots,N, s = PH, OH \qquad (60)$$

According to condition 2, each investment firm maximizes its profit, taking its own technology and the transmission price on each line as given: it chooses $\left(I_1^{r*},\dots,I_L^{r*}\right)$ so as to solve

$$\max_{I_\ell^r \ge 0, \forall \ell \in L} \sum_{s=PH,OH} \left(\sum_{\ell \in L} t_\ell^s I_\ell^r - w_s C^r\left(I_1^r,\dots,I_L^r\right)\right)$$

The corresponding necessary and sufficient conditions are:

$$\sum_{s=PH,OH} t_\ell^s \le \frac{\partial C^r}{\partial I_\ell^r}\left(I_1^{r*},\dots,I_L^{r*}\right) \sum_{s=PH,OH} w_s, \text{ with equality if } I_\ell^{r*} > 0, \forall \ell \in L \qquad (61)$$

In condition 3, each consumer maximizes her expected utility within the budget constraint, given the nodal price of her own node: she solves the following problem

$$\max_{\substack{\left(m_n^s, c_n^s\right) \in R \times R_+, \\ \forall s = PH, OH}} \sum_{s=PH,OH} \pi_s \left(m_n^s + \phi_n^s\left(c_n^s\right)\right)$$

$$\text{s.t.} \sum_{s=PH,OH}\left(w_s m_n^s + p_n^s c_n^s\right) \le \sum_{s=PH,OH}\left(\begin{array}{l} w_s m_n^s + \sum_{j=1}^{N}\theta_n^j\left(p_j^s q_j^{s*} - w_s z_j^{s*}\right) + \\ \sum_{r\in E}\theta_n^r\left(\sum_{\ell\in L} t_\ell^s I_\ell^{r*} - w_s z^{r*}\right) + \theta_n^T \sum_{\ell\in L} t_\ell^s k_\ell^0 \end{array}\right)$$

where the right-hand side of the constraint is consumer n's total wealth across the two states. The necessary and sufficient conditions for a utility maximizing consumption bundle are

$$\pi_s - \eta_n w_s = 0, s = PH, OH \tag{62}$$

$$\pi_s \frac{\partial \phi_n^s}{\partial c_n^s}\left(c_n^{s*}\right) \le \eta_n p_n^s, \text{ with equality if } c_n^{s*} > 0, \forall n = 1,\ldots,N, s = PH, OH \tag{63}$$

where η_n is the Lagrangian multiplier of the consumer's budget constraint. From equations (62) we have $\frac{\pi_{PH}}{\pi_{OH}} = \frac{w_{PH}}{w_{OH}}$. Since only the relative prices matter in deriving the equilibrium, we can let $w_{PH} = \pi_{PH}$ and $w_{OH} = \pi_{OH}$. Then $\eta_n = 1$ and (63) become

$$\pi_s \frac{\partial \phi_n^s}{\partial c_n^s}\left(c_n^{s*}\right) \le p_n^s, \text{ with equality if } c_n^{s*} > 0, \forall n = 1,\ldots,N, s = PH, OH \tag{64}$$

Besides, (60) and (61) can be rewritten as follows, respectively

$$p_n^s \le \pi_s \frac{\partial C_n^s}{\partial q_n^s}\left(q_n^{s*}\right), \text{ with equality if } q_n^{s*} > 0, \forall n = 1,\ldots,N, s = PH, OH \tag{65}$$

$$\sum_{s=PH,OH} t_\ell^s \le \frac{\partial C^r}{\partial I_\ell^r}\left(I_1^{r*},\ldots,I_L^{r*}\right), \text{ with equality if } I_\ell^{r*} > 0, \forall \ell \in L \tag{66}$$

We can infer from Section 3 that a necessary and sufficient condition for the no-arbitrage condition to hold is that in each state the price at the residual node N must be equal to the nodal price at each node $n=1,\ldots,N$ plus the transmission charge for transmitting one unit of power from node n to node N. Formally, conditions (58) are equivalent to

$$p_N^s = p_n^s + \sum_{\ell\in L} t_\ell^s \alpha_n^\ell\left(\left(I_v^*\right)_{v\in L}\right), \forall n = 1,\ldots,N-1, s = PH, OH \tag{67}$$

The proof of necessity and sufficiency is similar to that in Subsection 3.5 and will not be repeated here.

The last condition in the definition states that at the equilibrium prices, supply equals demand for the numeraire, power and transmission, respectively.

In summary, the necessary and sufficient conditions for allocation $\left(\left(\left(m_n^{s*},c_n^{s*}\right)_{n=1}^N,\left(-z_n^{s*},q_n^{s*}\right)_{n=1}^N\right)_{s=PH,OH},\left(-z^{r*},\left(I_\ell^{r*}\right)_{\ell\in L}\right)_{r\in E}\right)$ and price vector $(w,p,t)=\left(w_s,\left(p_n^s\right)_{n=1}^N,\left(t_\ell^s\right)_{\ell\in L}\right)_{s=PH,OH}$ to constitute a competitive equilibrium are:

$$p_n^s \le \pi_s \frac{\partial C_n^s}{\partial q_n^s}\left(q_n^{s*}\right), \text{ with equality if } q_n^{s*}>0, \forall n=1,\ldots,N, s=PH,OH$$

$$\pi_s \frac{\partial \phi_n^s}{\partial c_n^s}\left(c_n^{s*}\right)\le p_n^s, \text{ with equality if } c_n^{s*}>0, \forall n=1,\ldots,N, s=PH,OH$$

$$\sum_{s=PH,OH} t_\ell^s \le \frac{\partial C^r}{\partial I_\ell^r}\left(I_1^{r*},\ldots,I_L^{r*}\right), \text{ with equality if } I_\ell^{r*}>0, \forall \ell\in L$$

$$p_N^s = p_n^s + \sum_{\ell\in L} t_\ell^s \alpha_n^\ell\left(\left(I_v^*\right)_{v\in L}\right), \forall n=1,\ldots,N-1, s=PH,OH$$

$$\sum_{n=1}^N m_n^{s*} + \sum_{n=1}^N z_n^{s*} + \sum_{r\in E} z^{r*} = \varpi_s$$

$$\sum_{n=1}^N c_n^{s*} = \sum_{n=1}^N q_n^{s*}$$

$$\sum_{n=1}^{N-1} \alpha_n^\ell\left(\left(I_v^*\right)_{v\in L}\right)\left(q_n^{s*}-c_n^{s*}\right)\le k_\ell^0 + I_\ell^*, \text{ with equality if } t_\ell^s>0, \forall \ell\in L$$

At an Arrow-Debreu equilibrium all trade takes place simultaneously and before the uncertainty is resolved. In reality, trade takes place to a large extent sequentially over time, and frequently as a consequence of information disclosures. Mas-Colell, Whinston and Green (1995) have shown that Arrow-Debreu equilibria can be reinterpreted by means of trading processes that actually unfold through time. In this paper, we only adopt the Arrow-Debreu framework and do not consider sequential trading, but the result should be the same.

(4) Comparison between Arrow-Debreu equilibrium and efficient allocation

Now we want to answer these questions for the scenario with uncertainty: is the Arrow-Debreu equilibrium efficient or can the efficient allocation be decentralized? To be more specific, is the transmission investment level in equilibrium socially optimal? If not, what causes the problem? Recall that for the economy without uncertainty, the market equilibrium is not necessarily efficient, due to the externalities created by loop flows. Intuitively, this result should still hold here, since the inclusion of uncertainty does not change the fact that transmission investment may affect the flow of power along the lines for any given set of injections.

We observe that if for each two lines $\ell, \ell' \in L$, and for each node $n \in N$, investment in line ℓ does not affect the distribution factor $\alpha_n^{\ell'}$, that is, if $\frac{\partial \alpha_n^z\left(\left(I_v^*\right)_{v\in L}\right)}{\partial I_\ell} = 0$, then every efficient allocation can be decentralized by competitive prices, and the allocation in every Arrow-Debreu equilibrium is efficient. We can see that if an allocation $\left(\left(\left(c_n^{s*}, q_n^{s*}\right)_{n=1}^N\right)_{s=PH,OH}, \left(\left(I_\ell^{r*}\right)_{\ell\in L}\right)_{r\in E}\right)$ together with the Lagrangian multipliers λ_s and μ_ℓ^s for $\ell \in L$ and $s = PH, OH$ solve the social optimum problem (55), then $p_N^s = \lambda_s$, $p_n^s = \lambda_s - \sum_{\ell\in L} \mu_\ell^s \alpha_n^\ell$ for $n = 1,..., N$ and $s = PH, OH$, and $t_\ell^s = \mu_\ell^s$, for $\ell \in L$ and $s = PH, OH$ are the nodal and transmission prices that support the corresponding efficient allocation. Conversely, if $(w, p, t) = \left(w_s, \left(p_n^s\right)_{n=1}^N, \left(t_\ell^s\right)_{\ell\in L}\right)_{s=PH,OH}$ are prices that together with $\left(\left(\left(c_n^{s*}, q_n^{s*}\right)_{n=1}^N\right)_{s=PH,OH}, \left(\left(I_\ell^{r*}\right)_{\ell\in L}\right)_{r\in E}\right)$ constitute a competitive equilibrium, then $\lambda_s = p_N^s$, and $\mu_\ell^s = t_\ell^s$ together with $\left(\left(\left(c_n^{s*}, q_n^{s*}\right)_{n=1}^N\right)_{s=PH,OH}, \left(\left(I_\ell^{r*}\right)_{\ell\in L}\right)_{r\in E}\right)$ are a solution to the social optimum problem (55).

However, in general the distribution factors are affected by investment in different lines. Hence the flow structure of the network is changed after the investment, as indicated by the term $B = \sum_{s=PH,OH} \sum_{z=1}^{L} \mu_z^s \sum_{n=1}^{N-1} \frac{\partial \alpha_n^z\left(\left(I_v^*\right)_{v\in L}\right)}{\partial I_\ell}\left(q_n^{s*} - c_n^{s*}\right)$. This is the externality that causes the competitive allocations to be inefficient and the result that efficient allocations cannot be decentralized by means of competitive prices. So, we end up with the same conclusion as that from the model without uncertainty: the externality that causes market failure in the transmission investment market comes from the dependence of the distribution factors on investment. This externality exists as along as the investment affects the flow of power along the lines for a given set of injections. Obviously, the introduction of uncertainty does not change the basic result. Similarly, we can restore efficiency by imposing a unit tax of

$\tau_\ell^s = \sum_{z=1}^{L} \mu_z^s \sum_{n=1}^{N-1} \frac{\partial \alpha_n^z\left(\left(I_v^*\right)_{v\in L}\right)}{\partial I_\ell}\left(q_n^{s*} - c_n^{s*}\right)$ or a subsidy of $s_\ell^s = -\sum_{z=1}^{L} \mu_z^s \sum_{n=1}^{N-1} \frac{\partial \alpha_n^z\left(\left(I_v^*\right)_{v\in L}\right)}{\partial I_\ell}\left(q_n^{s*} - c_n^{s*}\right)$ on investment in line ℓ in state s for $\ell \in L$ and $s = PH, OH$.

Like before, we can extend this model with uncertainty to incorporate the case of grid expansion. The results should be the same.

CHAPTER 3. TRANSMISSION INVESTMENT COST ALLOCATION WITHIN THE COOPERATIVE GAME FRAMEWORK

3.1 Introduction

In recent decades, the electricity market in the U.S. as well as in many other countries has experienced an unprecedented institution restructuring from heavy regulation to competition. The subject of transmission expansion is important and recognized as a complex problem in electricity restructuring (Hogan 2003). One of the major concerns is how transmission investment costs should be allocated. Due to economies of scale and network effects, there may be situations where many would benefit from a transmission expansion but no coalition is prepared to make the investment. In this case, a regulatory decision to approve the investment and allocate the costs is required. If no coalition of grid users were able to agree to pay for a grid expansion that appears to be beneficial for the system as a whole, any interested party could propose a project and an allocation of its costs among those grid users who would benefit (Hogan 1999). This paper gives some cost allocation options, motivated by cooperative games.

Cooperative game theory arises as a most convenient tool to solve cost allocation problems and its application to the electricity industry has been growing in the literature. Contreras, Klusch and Yen (1998) and Contreras and Wu (2000) proposed a decentralized coalition formation and cost allocation procedure for transmission expansion planning, using the bilateral Shapley value and kernel, respectively. Zolezzi and Rudnick (2002) developed a new allocation method among the electric market participants, which is based mainly on the responsibility of the agents in the physical and economic use of the network, their rational behavior, the formation of coalitions, and cooperative game theory resolution mechanisms. Following that, Zolezzsi and Rudnick (2003) presented a transmission cost allocation method, based on cooperative game theory and transmission network capacity use by consumers.

Different than previous work, this paper identifies the situation in which socially worthwhile transmission investment may not benefit every agent and applies cooperative game theory to the allocation of transmission investment costs, which, as mentioned earlier, is of high significance and broad interest. We choose the cooperative game theory framework, because its solution mechanisms behave well in terms of fairness, efficiency and stability, qualities required for the correct allocation of transmission investment costs. Basically, the problem of transmission investment cost allocation can be formulated to a cooperative game. Then the well-known game theoretic solution concepts, such as the Shapley value, core and nucleolus can be used to solve the associated game. From these solutions, we can derive the allocation rules for the original cost allocation problem. We consider the bid-based, security-constrained economic dispatch system, which is now widely practiced in different regions in the U.S.

The rest of this chapter is organized as follows. We start, in Section 3.2, with an example with a simple three-bus electricity transmission network, from which one can see how the bid-based, security-constrained system works and how comes the investment cost allocation problem. Section 3.3 defines the general electricity cost allocation problem and allocation rule. Two cost allocation rules are provided in Section 3.4, based on two important cooperative game theoretic solution concepts, namely the Shapley Value, and core, respectively. In Section 3.5, we give the relationship between the electricity cost allocation problem and the bankruptcy problem and propose a third allocation rule, using the concept of nucleolus. The last section is a brief summary of earlier conclusions and suggests potential research extensions.

3.2 A Three-bus Example

Let us start from an example with a three-bus transmission network as illustrated in Fig. 1. We focus on the day-ahead market corresponding to the specified hour h and suppress the hour h in our notation. The basic setting is as follows: The network consists of three interconnected nodes. The impedance is the same for all the three lines. For simplicity, assume that only the line connecting nodes 1 and 3 is constrained to carry no more than 200 MW. The other two lines, in contrast, have extremely large capacities that they are never congested. Besides, assume that there is no voltage or thermal loss during the transmission. At each of the three nodes, there is a generator (or seller) that produces and sells electricity and a consumer (or buyer) that buys and consumes electricity[16]. We use S_n for the seller and B_n for the buyer, at node n, for $n = 1,2,3$. This transmission system is operated and supervised by an independent system operator (ISO).

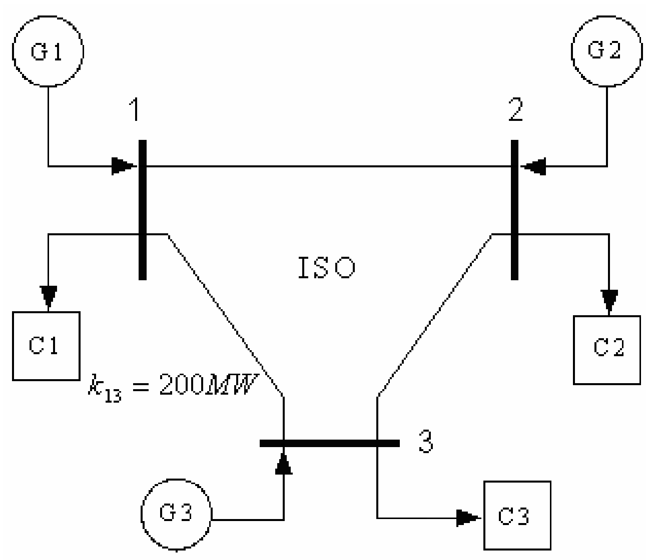

Figure 1: A three-bus network

Without loss of generality, we assume that the ISO is also the transmission owner (TO) of the transmission grid. The sellers and buyers in the market submit to the ISO sealed offers and bids, respectively, describing the price and quantity at which they are willing to sell or buy energy. The ISO determines the successful offers and bids and the market-clearing price by maximizing the social surplus, allowing for the physical constraints. The auction results determine the unit commitment and dispatch of the physical units. The offers submitted by the three sellers are illustrated in Figures 2-4, respectively.

The supply curve in Figure 2 means that the seller at node 1 can produce up to 300 MWh at a cost of $5 per MWh and another 300 MWh at a cost of $10. Similar interpretations apply to the other two supply curves. Figures 5-7 describe the bids of the three consumers, respectively:

[16] By the consumer, we actually mean any entity that purchases electricity from the system, such as the load serving entity (LSE).

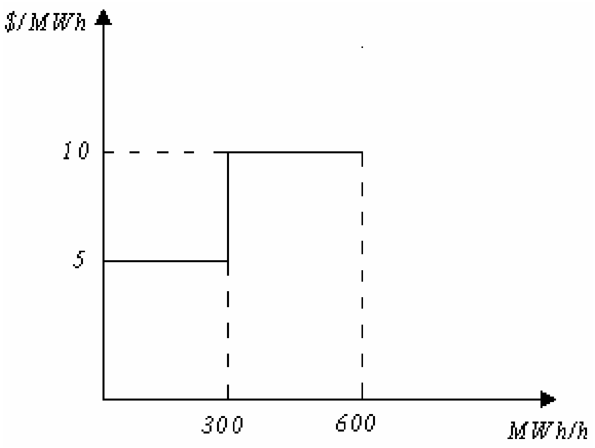

Figure 2: S1

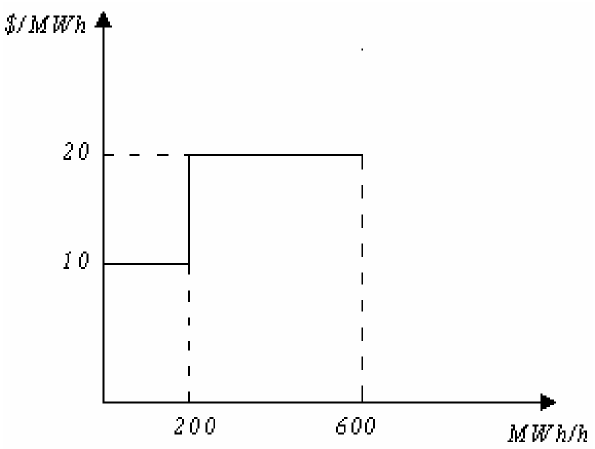

Figure 3: S2

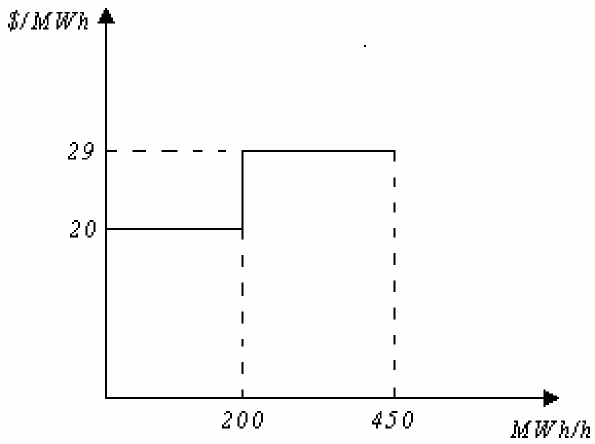

Figure 4: S3

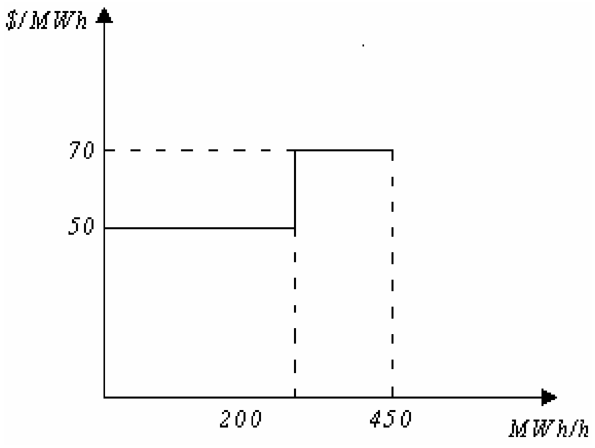

Figure 5: B1

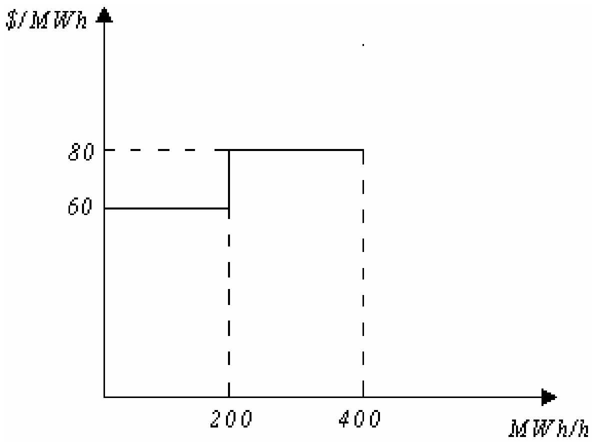

Figure 6: B2

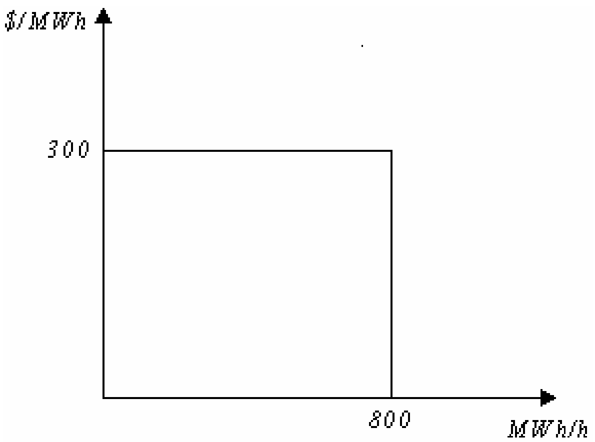

Figure 7: B3

For example, the demand curve in Figure 5 tells that buyer 1 is willing to pay $70 per MWh for up to 200 MWh and $50 per MWh for 100 more MWh. The other two demand curves can be interpreted in a similar way. The ISO maximizes the social surplus, given these offers and bids and the non-negativity, balancing and transmission constraints. In what follows, we consider both the unconstrained and constrained case.

3.2.1 Unconstrained Case

If there were no transmission constraint, the ISO solved the following problem

$$\max_{\substack{q_n \geq 0, c_n \geq 0 \\ \forall n=1,\ldots,N}} \sum_{n=1}^{N} U_n(c_n) - \sum_{n=1}^{N} C_n(q_n)$$

$$\text{s.t. } \sum_{n=1}^{N} c_n = \sum_{n=1}^{N} q_n \quad \text{(supply-demand balance)}$$

where $U_n(c_n)$ is buyer n's benefit or utility from consuming c_n amount of power and $C_n(q_n)$ is seller n's cost of generating q_n amount of power. Thus, the maximand $\sum_{n=1}^{N} U_n(c_n) - \sum_{n=1}^{N} C_n(q_n)$ is the social surplus, namely the difference between the social benefit and the social cost. The social surplus is maximized subject to the non-negativity and balancing constraints. Solving this problem for the three-bus example previously, we get the following generation and consumption quantities (in MWh):

$$q_1 = 600, q_2 = 600, q_3 = 300, c_1 = 300, c_2 = 400, c_3 = 800$$

The market clearing price is 29$/MW, which is the same at all busses. This price can be derived as the Lagrangian multiplier of the supply-demand balancing constraint. The social welfare is $SW_{uc} = 265{,}600$ (in $), in which the producer surpluses (or profits) for sellers 1, 2 and 3 are (in $): $PS_{1,uc} = 12{,}900$, $PS_{2,uc} = 7{,}400$ and $PS_{3,uc} = 1{,}800$, respectively and the consumer surpluses for buyers 1, 2 and 3 are (in $) : $CS_{1,uc} = 10{,}300, CS_{2,uc} = 16{,}400$, and $CS_{3,uc} = 216{,}800$, respectively.

3.2.2 Constrained Case

Now there is a flow constraint on line 1-3, i.e. the flow along this line can not exceed 200 MW. Under the unconstrained transmission dispatch, the power flow on Line 1-3 is 266.67 MW, which violates the transmission constraint. Therefore, the dispatch described in 3.2.2 is not feasible in reality and the ISO has to take the line constraint into account when solving for the optimal dispatch. Therefore, the ISO solves the following problem:

$$\max_{\substack{q_n \geq 0, c_n \geq 0 \\ \forall n=1,\ldots,N}} \sum_{n=1}^{N} U_n(c_n) - \sum_{n=1}^{N} C_n(q_n)$$

$$\text{s.t. } \sum_{n=1}^{N} c_n = \sum_{n=1}^{N} q_n$$

$$f_\ell(q_1,\ldots,q_N; c_1,\ldots,c_N) \leq k_\ell, \ \forall \text{ line } \ell \in L$$

Solving this new problem for our three-bus example, we get the quantities in the constrained case:

$$q_1 = 550, q_2 = 500, q_3 = 450, c_1 = 300, c_2 = 400, c_3 = 800$$

The constraint on line 1-3 is binding. In other words, line 1-3 is congested, which leads to different locational marginal prices (LMPs) at different nodes[17]. Specifically, the LMPs at node 1, 2 and 3 are 10, 20 and 30 (in \$/MW), respectively[18]. Note that electricity at each node is bought and sold at the LMP of that node. Therefore, sellers 1 and 2 receive \$10 and \$20, respectively for each MWh sold, and buyer 3 has to pay \$30 for each MWh power consumed, regardless of where the power comes from. As a result, the total amount paid by the consumers exceeds the total amount received by the sellers. The difference between these two amounts is known as the congestion rent (denoted by *K*), collected by the ISO. In this case, the social welfare is divided into three components: consumer surpluses, producer surpluses and congestion rent, which take the following values (in \$):

[17] The LMP at any location is the incremental cost of serving one more increment (1 MW) of load at each location, given the actual dispatch, the constraints, and the participants' offers/bids

[18] Let λ and μ be the Lagrangian multiplier of balancing and flow constraint, respectively. Then the LMP at node 3 is equal to λ and the LMPs at node 1 and 3 are $\lambda - 2\mu/3$ and $\lambda - \mu/3$, respectively.

$$PS_{1,c} = 1{,}500, PS_{2,c} = 2{,}000, PS_{3,c} = 2{,}250$$
$$CS_{1,c} = 16{,}000, CS_{2,c} = 20{,}000, CS_{3,c} = 216{,}000$$
$$K = 6{,}000$$

Adding them up, we have that the social surplus in the constrained case (SW_c) is \$263,750, which is \$1850 less than that in the unconstrained case. It means that the social surplus will increase by \$1850 if the constraint is eliminated[19]. So, if the cost of eliminating the constraint is lower than \$1,850, it is socially worthwhile to make the transmission investment that removes the constraint.

The agents will benefit or suffer differently from the investment, although it might be good for society as a whole. From now on, we will change the indices for notational simplicity: 1—S_1, 2—S_2, 3—S_3, 4—B_1, 5—B_2, 6—B_3, and 7—ISO. Let π_i, for $i = 1,...,7$ be the change in agent *i*'s welfare from the constrained to the unconstrained case. Then we have (in \$):

$$\pi_1 = 11400, \pi_2 = 5400, \pi_3 = -450, \pi_4 = -5700, \pi_5 = -3600, \pi_6 = 800, \pi_7 = -6000$$

These numbers say that if the constraint on line 1-3 is eliminated through transmission investment, sellers 1 and 2 and buyer 3 will be better off, while the other four agents, worse off. This three node example can be generalized to one with a larger, more complicated set of nodes and more agents. An investment project that enhances transmission capacity of the network may benefit society as a whole, but not every agent. Then a series of questions ensure. Who should make the beneficial investment? How should the investment cost be allocated? Intuitively, the beneficiaries should finance the investment and compensate the sufferers for their loss. In the following sections, we will formulate the situation into a cost allocation problem and characterize its allocation rules.

3.3 Electricity Cost Allocation Problem and Allocation Rule

For a given transmission network and a set $N = \{1,2,...,n\}$ of agents, we can define the problem of sharing or allocating transmission investment costs as follows:

Definition 1: An electricity cost allocation problem is a pair (C, π), where

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[19] Here we ignore the effect of capacity enhancement in line 1-3 has on the impedance of that line, nor the distribution factors of the network. Hence, the line impedances and distribution factors remain the same before and after the investment.

$\pi = (\pi_1, \pi_2, ..., \pi_n) \in R^n$ and $0 \le C \le \sum_{i=1}^{n} \pi_i$.

Here C is the cost of the transmission investment project and π_i , for $i = 1,...,n$ is agent i's benefit from the investment. Note that π_i can be negative, meaning that agent i will suffer, rather than benefit from the investment.

An *allocation* in the problem (C, π) is a vector $x = (x_1,..., x_n)$, such that $\sum_{i=1}^{n} x_i = C$. The value x_i is interpreted as agent i's contribution to the financing of the cost C. An allocation is said to be *individually rational* if $x \le (\pi_1,..., \pi_n)$. The requirement $x_i \le \pi_i$ for $i = 1,...,n$ means that if agent i benefits from the investment, her contribution should be no greater than her benefit; if she suffers from the investment, she has to be compensated for her loss. Given an allocation x, the corresponding vector of net benefit is $\pi - x$.

Our task is to find reasonable cost allocations. Specifically, we are interested in reasonable allocation rules that associate an allocation to each electricity cost allocation problem.

Definition 2: An allocation rule is a function f that maps each electricity cost allocation problem (C, π) to one or a set of its allocations.

That is, given an electricity cost allocation problem, our rule should tell us how much each agent will end up paying. One of the possible rules can be the following:

Example: The Head Tax Rule is the rule that maps each allocation problem (C, π) to the allocation *hta* (C, π) = x where $x_i = \min\{\lambda, \pi_i\}$, $\forall i = 1,...,n$ and $\lambda \ge 0$ is chosen such that $\sum_{i=1}^{n} x_i = C$. This allocation rule dictates that those whose welfare will be hurt by the investment should be fully compensated for their losses ($x_i = \pi_i$, if $\pi_i < 0$), and those who will be better-off after the investment should contribute an equal amount λ to its financing, as long as no one pays more than her benefit, in which case she pays her full benefit.

For the purpose of finding allocation rules, we will translate the electricity cost allocation problem into a cooperative game with transferable utility and then apply the well-known game-theoretic solution concepts to the game to get allocation rules for the original problem.

Definition 3: A cooperative game (or a game in coalitional form) consists of

1. a set N (the players), and
2. a function $v: 2^N \to R$, such that $v(\phi) = 0$ $(2^N = \{S : S \subseteq N\})$.

A subset S of N is called a *coalition*, and $v(S)$ is called the *worth* of the coalition S. Intuitively, $v(S)$ represents the total amount of payoff that the coalition S can get by itself, without the help of the other players. For the electricity cost allocation scenario, we assume that each coalition can carry out the investment project as long as it pays the investment cost and compensates all the agents that will suffer from the project. The idea is that each sufferer has the veto power, unless she is fully compensated for her loss. Now we can formulate the cooperative game from the electricity cost allocation problem.

Definition 4: Let (C, π) be an electricity cost allocation problem defined in Definition 1. The associated cooperative game is (N, v), where

1. $N = \{1, \ldots, n\}$ is the set of players, and
2. $v(S) = \max\{0, \sum_{i \in S} \pi_i - C + \sum_{i \notin S} \min\{0, \pi_i\}\}, \quad \forall S \subseteq N$.

Condition 2 above means that a coalition S can always remain in the status quo, and hence get 0, or undertake the transmission investment after compensating the agents outside the coalition for the losses they might suffer.

A *payoff vector* of the game is a vector $y = (y_1, \ldots, y_n)$ with the element y_i representing the payoff to player i or equivalently what player i will finally get. It will be recalled that the set of *imputations* of a cooperative game (N, v) is the set of payoff vectors y, such that $y \geq 0$ and $\sum_{i \in N} y_i = v(N)$. Since $v(N) = \max\{0, \sum_{i \in N} \pi_i - C\}$ in our game, its set of imputations is the set of vectors $y \geq 0$, such that $\sum_{i \in N} y_i = \max\{0, \sum_{i \in N} \pi_i - C\}$. Every imputation y of the game (N, v) induces a cost allocation for the electricity cost allocation problem (C, π) in the way that $x = \pi - y$.

Having the definitions above, we can apply some accepted game theoretic solution concepts to the game induced by the cost allocation problem and obtain the corresponding cost allocations. Solution concepts associate payoff vectors with games. In many cases a solution concept associates several payoff vectors with a given game or none at all.

3.4 Allocation Rules Based on the Shapley Value and Core

In this section, we will find the allocation rules, using two game theoretic solution concepts, namely the Shapley value and the core. The former is single-valued and associates a unique payoff vector with each cooperative game. Let us first recall the definition of the Shapley value.

Definition 5: Let $N = \{1,2,...,n\}$ and G^N be the set of all games whose player set is *N*. The Shapley value or value on N is a function $\phi : G^N \to R^N$ that satisfies the following conditions:

1. (Symmetry condition): if *i* and *j* are substitutes[20] in *v*, then $(\phi v)_i = (\phi v)_j$.
2. (Null player[21] condition): if *i* is a null player, then $(\phi v)_i = 0$.
3. (Efficiency condition): $\sum_{i=1}^{n} (\phi v)_i = v(N)$.
4. (Additivity condition): $(\phi(v+w))_i = (\phi v)_i + (\phi w)_i$.

$(\phi v)_i$, the *i*-th coordinate of the image vector $\phi(v)$ is interpreted as the "power" of player *i* in the game *v*, or what it is worth to *i* to participate in the game. The four conditions that the Shapely value must satisfy are quite intuitive. Condition 1 necessitates that if two players have equal influence on any coalition without either of them originally, their payoff should be the same. Naturally, a player that has no influence at all should get nothing, which is what Condition 2 means. The efficient condition in 3 dictates that the total benefit should be fully distributed among the players. The last condition says that the Shapley value is an additive allocation rule. The Shapley value calculates a fair division of the utility, based on individuals' contributions, among the members in a coalition. It can be considered as a

[20] Substitutes: *i* and *j*, elements of *N*, are substitutes in *v* if for all *S* containing neither *i* nor *j*, $v(S \cup \{i\}) = v(S \cup \{j\})$.

[21] An element *i* of *N* is called a null player if $v(S \cup \{i\}) = v(S)$ for all $S \subseteq N$.

weighted average of marginal contributions of a member to all the possible coalitions in which it may participate.

For every *N*, there exists a unique Shapley value on G^N, which can be calculated using the formula below.

Theorem 1 (Shapley [1953])**:** $(\varphi v)i = (1/n!)\sum_R [v(S_i \cup \{i\} - v(S_i)]$, where *R* runs over all *n!* different orders on *N*, and S_i is the set of players preceding *i* in the order *R*.

This theorem enables us to solve for the Shapley value for any given cooperative game, although the calculation might be complicated sometimes. In fact, in some cases, the value can be more easily found using the definition itself, as we will see later.

Another important concept of the solution to the cooperative game is the core, which is defined as follows.

Definition 6: The core of the game (*N, v*) is the set of all imputations *y*, such that $V(S) \le \sum_{i \in S} y_i$ for all $S \subset N$.

Unlike the Shapley Value the core is not a single payoff vector, but a set of payoff vectors, which can be empty. The core of a cooperative game consists of all undominated allocations in the game. In other words, the core includes all allocations with the property that no group can do better by deserting the grand coalition.

There is no general relationship between the two solution concepts described so far. The Shapley value may not be in the core, even when it is not empty. Intuitively the Shapley Value represents a reasonable compromise, whereas the core represents a set of payoff vectors which are, in a certain sense, stable.

For each electricity cost allocation problem (C, π), we can calculate the Shapley value and the core of the associated cooperative game (*N, v*). Then an allocation to the original electricity problem can be $x = \pi - y$ where y is the Shapley value or an imputation vector in the core, depending on which solution concept we use.

As an application, we adopt the method to the electricity cost allocation problem (C, π) formulated from our three-bus example, where $C \ge 1850$, $\pi_1 = 11400$, $\pi_2 = 5400$, $\pi_3 = -450$, $\pi_4 = -5700$, $\pi_5 = -3600$, $\pi_6 = 800$, and $\pi_7 = -6000$. The associated cooperative game is (*N, v*), where $N = \{1,2,3,4,5,6,7\}$, and

$$\begin{aligned} v(S) &= \max\{0, 11400 + 5400 + 800 - 15750 - C\} \\ &= \{0, 1850 - C\} = 1850 - C \\ &\forall S \subseteq N, \text{s.t.} 1, 2, 6 \in S \end{aligned} \quad (1)$$

$$\begin{aligned} v(S) &= \max\{0, 11400 + 5400 - 15750 - C\} \\ &= \{0, 1050 - C\} \\ &\forall S \subseteq N, \text{s.t.} 1, 2 \in S, \text{and } 6 \notin S \end{aligned} \quad (2)$$

$$v(S) = 0 \ \ \forall \text{ other } S \subseteq N \quad (3)$$

3.4.1 Allocation Rule Based on the Shapely Value

For calculation simplicity, let us further assume that $C < 1050$. Since players 3, 4, 5 and 7 are null players in this game, their powers are all 0, i.e. $(\phi v)_3 = (\phi v)_4 = (\phi v)_5 = (\phi v)_7 = 0$. Players 1 and 2 are substitutes, so $(\phi v)_1 = (\phi v)_2$. By Theorem 1, we have that the power of buyer 3 is $(\phi v)_6 = \frac{800}{3}$. Then $(\phi v)_1 = (\phi v)_2 = \frac{(v(N) - (\phi v)_6)}{2} = \frac{2,375}{3} - \frac{C}{2}$. The corresponding allocation to the original electricity problem can then be derived as $x_i = \pi_i - y_i$ for $i \in N$ and has the following values:

$$x = \{\frac{31825}{3} + \frac{C}{2}, \frac{13825}{3} + \frac{C}{2}, -450, -5700, -3600, \frac{1600}{3}, -6000\}$$

At this allocation, sellers 1 and 2 and buyer 3 share the investment cost and fully compensate the others for their losses. Moreover, the more does one benefit from the investment, the more she would pay. Therefore, among the three beneficiaries, seller 1 pays the most, and buyer 3, the least.

3.4.2 Allocation Rule Based on the Core

Let $\{ y_{1,} y_2, y_3, y_4, y_5, y_6, y_7 \}$ be a payoff vector for the players of the game. For it to be in the core, the payoff vector should satisfy the conditions specified in Definition 6, which can be simplified to the following:

$$y_{1,} y_2, y_3, y_4, y_5, y_6, y_7 \geq 0$$

$$y_1 + y_2 \geq v(1, 2) = 1050 - C$$

$$y_1 + y_2 + y_6 \geq v(1, 2, 6) = 1850 - C$$

$$y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 = v(N) = 1850 - C$$

Then we get:

$$y_3 = y_4 = y_5 = y_7 = 0,\ y_6 \leq 800 \text{ and } y_1 + y_2 + y_6 = 1850 - C$$

Therefore, the core of the game is $\{a, b, 0, 0, 0, c, 0\}$, where $0 \leq c \leq 800$, $a + b = 1850 - C - c$, and $a, b \geq 0$.

Because $x = \pi - y$, an allocation rule of this electricity cost allocation problem based on the core is given as follows:

$$x = \{x_1, x_2, -450, -5700, -3600, x_6, -6000\}$$

where $14{,}950 + C \leq x_1 + x_2 \leq 15{,}750 + C$ and $0 \leq x_6 \leq 800$. As is in 3.4.1, the investment cost and compensation for the sufferers should be allocated among S_1, S_2, and B_3 only. The most noticeable difference between the allocation rules in 3.4.1 and 3.4.2 is that the rule in 3.4.1 maps the electricity problem to one single allocation, while 3.4.2 gives a set of allocations. In our example, the allocation based on the Shapley value is included in the set of allocations based on the core. Actually, the Shapley value is always in the core for the type of games we study in this paper, but it is not necessarily true for any arbitrary game[22].

More generally, for any electricity cost allocation problem, the allocation rules based on the Shapley Value and core both dictate that the agents that will benefit from the investment project should undertake the investment, finance it and compensate those who will suffer from it. The potential sufferers should be fully compensated such that they are neither better-off nor worse-off after the investment.

[22] As is shown in 3.5, the electricity cost allocation problem is a bankruptcy problem, for which the Shapley value is always in the core.

3.5 Bankruptcy Problem and Allocation Rule Based on the Nucleolus

So far, we have formulated the cooperative game associated with the electricity cost allocation problem and defined the allocation rules based on the Shapley value and core of the game. In this section, we find a third allocation rule by applying another solution concept, the nucleolus to the induced cooperative game. Like the Shapley value, the nucleolus is also a one point solution. Here is the interpretation behind the notion of the nucleolus. Given a payoff vector y each coalition S looks at $v(S)-y(S)$, which represents the "complaint" of the coalition (it could be positive or negative). The higher the complaint the more strongly the coalition objects to y. Thus we want to minimize complaints under the feasibility constraint. We do so starting with the maximal complaint, i.e., we look at $\min_y\left\{\max_{S\subseteq N}(v(S)-y(S))\right\}$. Then we minimize the next highest complaint when considering only the payoff vector that minimized the highest complaint, and so on. What we get is the lexicographic minimum of all complaints. It turns out that we are left with a unique payoff vector which is the nucleolus.

The nucleolus has the virtue that it is always in the core when the core is non-empty. The drawback is that the calculation of the nucleolus is extremely complicated in general. Fortunately, the nucleolus is easy to find for some types of cooperative games. One of them is the game induced by the *bankruptcy problem*, whose definition is given below. Although its motivation is different, we will see that the bankruptcy problem and the electricity cost allocation problem are closely related.

3.5.1 Bankruptcy Problem and Consistent Allocation

Definition 7 (Aumann and Maschler (1985))**:** A bankruptcy problem is defined as a pair (*E; d*), where $d=(d_1,\ldots,d_n)$, $0\le d_1\le\ldots\le d_n$ and $0\le E\le d_1+\ldots+d_n$.

$E\in R$ is the estate and $(d_1,\ldots,d_n)\in R^n$ denotes a vector of claims or debts. The condition $0\le E\le d_1+\ldots+d_n$ means that the total value of the estate can not cover the total liability, hence the name "bankruptcy".

An *allocation* to the bankruptcy problem (*E; d*) is an n-tuple $y = (y_1, y_2, ..., y_n)$ of real numbers with $y_1 + y_2 + ... + y_n = E$. For each bankruptcy problem (*E; d*) there is an associated cooperative game (*N, w*), where $N = \{1,2,...,n\}$ and $w(S) = \max\left\{0, E - \sum_{i \notin S} d_i\right\}$.

The imputations to this game will give allocations to the corresponding bankruptcy problem.

First think of a two-player bankruptcy problem $(E; (d_1, d_2))$, One possible allocation can be found in the following way. Each claimant *i* concedes $\max\{0, E - d_i\}$ to the other claimant *j*. The amount at issue is therefore $E - \max\{0, E - d_1\} - \max\{0, E - d_2\}$; it is shared equally between the two claimants, and in addition, each claimant receives the amount conceded to her by the other one. Thus the total amount awarded to *i* is

$$y_i = \frac{E - \max\{0, E - d_1\} - \max\{0, E - d_2\}}{2} + \max\{0, E - d_j\}, \text{ for } i, j = 1,2 \text{ and } i \neq j$$

We will say that the above division of *E* for claims d_1 and d_2 is prescribed by the *Contested Garment (CG)* principle. If one views the allocation as a function of *E*, one obtains such a process: Let $d_1 \leq d_2$. When *E* is small, it is divided equally. This continues until each claimant has received $d_1 / 2$. Each additional dollar goes to the greater claimant, until each claimant has received all but $d_1 / 2$ of her claim. Beyond that, each additional dollar is again divided equally.

The CG principle generalized to a rule for the *n*-player bankruptcy problem $(E; (d_1, ..., d_2))$ is called the *coalitional procedure*. Let $D = \sum_{i \in N} d_i$ and assume that $d_1 \leq d_2 \leq ... \leq d_n$. According to the coalitional procedure, we treat a given *n*-person problem in one of the following three ways, depending on the values of *E* and *d*:

1. Divide *E* between {*1*} and {*2, …, n*} in accordance with the CG solution of a 2-person problem, and then use the (*n-1*)-person rule, to divide the amount assigned to the coalition {*2, …, n*} between its members.
2. Assign equal awards to all creditors.
3. Assign equal losses to all creditors.

Specifically, 1 is applied when $\frac{nd_1}{2} \le E \le D - \frac{nd_1}{2}$; we use 2 when $E \le \frac{nd_1}{2}$ and 3 when $E \ge D - \frac{nd_1}{2}$.

This coalitional procedure (or CG principle for in 2-person case) generates an allocation to the bankruptcy problem. Given an allocation, if for all $i \ne j$, the division of $y_i + y_j$ prescribed by the contested garment (CG) principle for claims $d_{i,} d_j$ is (y_i, y_j), this solution is called a *consistent* solution. That is, at a consistent solution, the redistribution between any two agents according to CG principle will yield exactly the same result. Aumann and Maschler (1985) show that each bankruptcy problem has a unique consistent solution, which is exactly the *nucleolus* of the corresponding cooperative game. And the coalitional procedure described above computes this consistent solution, or the nucleolus of the induced game.

3.5.2 Electricity Cost Allocation Problem and Bankruptcy Problem

Now we have known much about the bankruptcy problem. In what follows, we will show that our electricity cost allocation problem is closely related to the bankruptcy problem, so that we can apply our knowledge of the bankruptcy problem to the electricity problem to define its allocation rule.

Observation: For each electricity cost allocation problem (C, π), there is a bankruptcy problem (*E; d*), such that both problems induce the same cooperative game.

Proof. Given (C, π), define a bankruptcy problem (*E; d*), where $E = \sum_{i=1}^{n} \pi_i - C$ and $d = (\max\{0, \pi_i\})_{i=1}^{n}$. Obviously the cooperative games induced by the two problems have the same set of players, so what remains is to prove that the worth of each coalition is the same for the two games. The game associated with (C, π) has the worth

$$v(S) = \max\{0, \sum_{i \in S} \pi_i - C + \sum_{i \notin S} \min\{0, \pi_i\}\}, \ \forall S \subseteq N \qquad (4)$$

The game associated with (*E; d*), where $E = \sum_{i=1}^{n} \pi_i - C$ and $d = (\max\{0, \pi_i\})_{i=1}^{n}$ has the worth

$$
\begin{aligned}
v(S) &= \max\{0, \sum_{i=1}^{n} \pi_i - C - \sum_{i \notin S} \max\{0, \pi_i\} \\
&= \max\{0, \sum_{i \in S} \pi_i - C + \sum_{i \notin S} \min\{0, \pi_i\} + \sum_{i \notin S} \max\{0, \pi_i\} - \sum_{i \notin S} \max\{0, \pi_i\}\} \qquad (5) \\
&= \max\{0, \sum_{i \in S} \pi_i - C + \sum_{i \notin S} \min\{0, \pi_i\}\}, \forall S \subseteq N
\end{aligned}
$$

The expression on the right of the last equation mark is exactly the same as the right-hand side in (4). Therefore, (C, π) and $(E;\ d)$ induce the same cooperative game. *(QED)*

Observation 1 is important, because it allows us to get allocations of the electricity cost allocation problems, a new type of problems by finding the solutions to the corresponding bankruptcy problems, which we are already very familiar with. According to this observation, each electricity cost allocation problem (C, π) is associated with a bankruptcy problem

$$(E;(d_{1,}\ldots,d_n)) \equiv (\sum_{i=1}^{n} \pi_i - C; (\max\{0, \pi_i\})_{i=1}^{n}) \qquad (6)$$

In this problem, the total surplus $\sum_{i=1}^{n} \pi_i - C$ has to be divided among the agents. Applying the coalitional procedure, we will get the consistent allocation y for this problem. Then the allocation of the original electricity problem can be derived as $x = \pi - y$.

3.5.3 Allocation Rule Based on the Nucleolus

As an example, consider the class of electricity problems $(C, (\pi_1, \pi_2))$ with only two agents. Without loss of generality, assume that $\pi_1 \geq \pi_2$. By the CG principle we get the allocation of surplus (y_1, y_2) for the problem in (6):

$$(\pi_1 - \frac{C}{2}, \pi_2 - \frac{C}{2}), \text{ if } \pi_1, \pi_2 \geq C$$

$$(\pi_1 - C + \frac{\pi_2}{2}, \frac{\pi_2}{2}), \text{ if } \pi_1 \geq C \text{ and } 0 \leq \pi_2 < C$$

$$(\pi_1 + \pi_2 - C, 0), \text{ if } \pi_1 \geq C \text{ and } \pi_2 < 0$$

$$(\frac{\pi_1 + \pi_2 - C}{2}, \frac{\pi_1 + \pi_2 - C}{2}), \text{ if } 0 \leq \pi_1, \pi_1 < C$$

Each pair of numbers in the parentheses above represent the allocation of the total surplus between the two players. Specifically, if both agents' individual benefits from the investment exceed the investment cost, the cost will be equally distributed between the agents and each agent will end up getting her original benefit less half of the investment cost. If both agents benefit from the investment, but one benefits more than the cost and the other, less than the cost, the latter will get half of her benefit and the rest will go to the pocket of the former. Another possible case is that one of the agents has a benefit higher than the investment cost, while the other agent will suffer from the investment. In that case, the beneficiary has to pay the investment cost and fully compensate the sufferer for her loss, such that the sufferer's welfare will be exactly the same as that before the investment. So the beneficiary will end up with all the social surplus. The social surplus will be equally divided between the two agents, if both of them benefit from the investment, but each has the benefit lower than the investment cost.

Having the allocation of surplus, we can obtain the cost allocation of the electricity problem:

$$(\frac{C}{2},\frac{C}{2}), \text{ if } \pi_1,\pi_2 \geq C;$$

$$(C-\frac{\pi_2}{2},\frac{\pi_2}{2}), \text{ if } \pi_1 \geq C \text{ and } 0 \leq \pi_2 < C;$$

$$(C-\pi_2,\pi_2), \text{ if } \pi_1 \geq C \text{ and } \pi_2 < 0;$$

$$(\frac{\pi_1-\pi_2+C}{2},\frac{\pi_2-\pi_1+C}{2}), \text{ if } 0 \leq \pi_1,\pi_2 < C.$$

Now we define the *electricity CG principle* for the 2-player electricity cost allocation problem:

1. If one player suffers from the transmission investment, e.g. $\pi_2 < 0$, she will receive the full amount of her loss π_2, or in other words, pay the negative amount of her loss. The investment cost plus the compensation for the sufferer $C-\pi_2$ will be defrayed by the beneficiary.

2. If both players benefit from the transmission investment, i.e. $\pi_1,\pi_2 \geq 0$, the cost that player i will pay is

$$x_i = \frac{C - \max(0, C - \pi_1) - \max(0, C - \pi_2)}{2} + \max(C - \pi_j)$$

for $i \neq j$. This formula dictates such a process: No one will pay more than her benefit, so each player i needs to pay at least $\max(0, C - \pi_j)$, imposed by the other player j. The amount at issue is therefore $C - \max(0, C - \pi_1) - \max(0, C - \pi_2)$.It is shared equally between the two players. Plus the amount imposed on her, the total cost player i has to pay will be $\frac{C - \max(0, C - \pi_1) - \max(0, C - \pi_2)}{2} + \max(C - \pi_j)$.

Note that the electricity CG principle described above is *monotonic*, in the sense that for fixed benefits (or losses) π_1, π_2, each of the two payments is non-decreasing in the investment cost C. That is, no one will end up paying less if the investment cost increases.

For the general n-player electricity problem, the cost allocation can be derived from $x = \pi - y$, where y is the consistent allocation of the corresponding bankruptcy problem in (6), or equivalently the nucleolus of the associated cooperative game. Let us call the rule the *electricity coalitional procedure*. We will show shortly that the allocation prescribed by this procedure is a consistent allocation in the electricity cost allocation context. That is, at this allocation $(x_1, ..., x_2)$, for all $i \neq j$, the division of $x_i + x_j$ determined by the electricity CG principle for claims π_i, π_j is still (x_i, x_j).

Proposition 1: The electricity coalitional procedure yields the consistent allocation for all electricity cost allocation problems.

Proof. Let $(x_1, ..., x_2)$ be the allocation prescribed by the electricity coalitional procedure of the electricity problem $(C, (\pi_1, ..., \pi_n))$. Let $(y_1, ..., y_n)$ be the consistent allocation of the corresponding bankruptcy problem $(\sum_{i=1}^{n} \pi_i - C; (\max\{0, \pi_i\})_{i=1}^{n})$. We must have $x_i = \pi_i - y_i$, for $i=1, ..., n$. We need to show that (x_i, x_j) is the allocation from the CG principle of the electricity problem $(x_i + x_j, (\pi_1, ..., \pi_n))$ for all $i \neq j$. Note this 2-player problem is equivalent to the 2-person bankruptcy problem $(y_i + y_j; ((\max(0, y_i), \max(0, y_j)))$. Consistency implies that the allocation to this problem dictated by the CG principle is

(y_i, y_j). So the allocation of the electricity problem $(x_i + x_j, (\pi_1, ..., \pi_n))$ prescribed by the electricity CG principle is (x_i, x_j). *(QED)*

Then does the electricity problem have a unique consistent allocation, as does the bankruptcy problem? The answer is yes.

Proposition 2: Each electricity cost allocation problem has a unique consistent allocation.

Proof. First we prove that there is at most one consistent solution. If there were more, we could find consistent solutions *x* and *z*, and players *i* and *j*, with $z_i > x_i$, $z_j < x_j$, and $z_i + z_j \geq x_i + x_j$. Consistency implies that if only *i* and *j* are involved, the CG principle allocates z_j to *j* when the total cost is $z_i + z_j$, and x_j when it is $x_i + x_j$. Since $z_i + z_j \geq x_i + x_j$, the monotonicity of the CG principle then implies $z_j \geq x_j$, contradicting $z_j < x_j$.

Second we show that there is at least one consistent solution. This holds, since we already know one, the allocation based on the nucleolus of the induced cooperative game. *(QED)*

Because the allocation based on the nucleolus of the corresponding cooperative game is consistent, and the electricity problem has only one consistent allocation, it is safe to say that the nucleolus-based allocation is the unique consistent allocation.

Now let us apply the rule to the three-bus cost allocation problem earlier. The problem is equivalent to a bankruptcy problem (*E; d*), where $E = 1850 - C \geq 0$ and d = {*11400, 5400, 0, 0, 0, 800, 0*}. The consistent solution to this bankruptcy problem is y = $\{y_1, y_2, y_3, y_4, y_5, y_6, y_7\}$, where $y_3 = y_4 = y_5 = y_7 = 0$; If $C \leq 650$, then $y_6 = 400$, $y_1 = y_2 = \frac{1450 - C}{2}$; If $C > 650$, then $y_1 = y_2 = y_6 = \frac{1850 - C}{3}$. So the allocation rule to the original electricity problem is $x = (x_1, x_2, ..., x_n)$, where $x_3 = -450$, $x_4 = -5700$, $x_5 = -3600$, and $x_7 = -6000$; and if $C \leq 650$, $x_1 = \frac{21350 - C}{2}$, $x_2 = \frac{9350 - C}{2}$, and $x_6 = 400$; if $C > 650$ $x_1 = \frac{32350 + C}{3}$, $x_2 = \frac{14350 + C}{3}$, and $x_6 = \frac{550 + C}{3}$.

Similar to the allocation rules defined in Section 3.4, this rule dictates that the agents who will suffer from the investment should be fully compensated and that all the cost of investment plus compensation should be shared by the agents who will benefit.

3.6 Conclusions

To sum up, we have defined rules for allocating electricity transmission investment cost on the basis of three different solution concepts of the cooperative game, namely the Shapley value, the core and the nucleolus. These rules give reasonable allocations for the electricity cost allocation problem. The central point in each rule is that the potential beneficiaries of transmission investment should pay to get benefits and the potential sufferers, be compensated for their losses. Note that the allocations rules we propose provide options for electricity cost allocation in the absence of any mechanism. They tell what we should do in theory, and hence provide a benchmark, against which the allocation methods in reality can be compared to see if they are proper or need to be improved upon. One can not say which rule is the best and which is the worst.

In this paper, we do not consider the possibility of the agents gaming against the system or any uncertainty. Nor do we incorporate the dynamic aspects such as the market entry in the long run. With these things incorporated, the calculation of the benefits and losses of transmission investment would be more complicated. But the allocation rules we derive here are still applicable, as long as we can figure out the benefits and losses.

The allocation rules presented in this paper also apply to the situation with uncertainty, as long as we can identify the benefit or loss from the transmission investment for each agent. The difference is that in the uncertainty case, expected benefits and losses rather than deterministic ones might be used. The methods for determining the expected benefits and losses are seen in the literature.

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CHAPTER 4. EVALUATING THE EFFICIENCY OF FINANCIAL TRANSMISSION RIGHTS AUCTIONS: EVIDENCE FROM THE U.S. MIDWEST ENERGY REGION

4.1 Introduction

In March 2005, the Midwest Independent System Operator (MISO) officially adopted the Wholesale Power Market Platform (WPMP) proposed by U.S. Federal Energy Regulatory Commission (FERC) in April 2003. One of the important features of FERC's WPMP design is to help alleviate the transmission congestion problems by issuing financial transmission rights (FTRs). By construction, an FTR is a financial contract that entitles the holder to a stream of revenues (or charges) based on the difference between the hourly day-ahead locational marginal price (LMP) at the sink and source nodes. Due to congestion on transmission lines, day-ahead LMPs can be very volatile, and FTRs make a hedging instrument against the price risks. In principle, market participants could reduce the price uncertainty by purchasing FTRs for a specified amount of MWs on the paths of the transmission grid that they anticipate to be congested during a given period of time[23].

But in real practice, to what extent FTRs have performed in helping market participants hedge transmission congestion exposure in the new Midwest wholesale power market is still unclear. Moreover, is the current FTRs market efficient in terms of having the clearing FTR prices match closely with agents' expectations about financial loss caused by transmission congestions? In this study, we will address these two questions using the publicly available MISO FTR auction data and historical LMP data.

As far as we know, no empirical work up to now has been done to analyze the MISO FTR market. Even for a broader geographic range, only a handful few studies have been conducted to investigate the empirical aspects of FTR market in other regions such as New York. Adamson and Englander (2005) examined the efficiency of the New York

[23] FTRs are available not only for physical paths. They can be defined between any two nodes in the grid.

transmission congestion contract (TCC) market[24]. They used monthly TCC auction prices and congestion revenues between November 1999 and April 2003. A two-stage modeling approach was adopted to analyze the data. In the first stage, they used the time series ARCH-ARMA model to forecast the mean and variance of spot prices (congestion rents). Then in the second stage, a simple linear model was proposed to regress TCC auction prices on the predicted mean and variance of spot prices from the first stage of modeling. From the results, they concluded that the New York TCC auctions were highly inefficient, even after allowing for risk aversion among bidders in the auctions.

Siddiqui et al. (2005) carried out another empirical study on the New York TCC market based on annual TCC auctions in years 2000 and 2001. They found that although TCCs appeared to provide a potentially effective hedge against volatile congestion rents, the prices paid for TCCs were systematically different from the resulting congestion rents. Their conclusion was that the unreasonably high risk premiums paid for the TCCs suggested an inefficient market and that the possible explanations were lack of liquidity in TCC markets and the difference between TCC feasibility requirements and actual energy flows. However, these results held only under the assumption that market participants are all risk-neutral. Risk-averse agents, instead, may pay for TCCs the amounts more than the expected congestion charges. Therefore, the deviation of TCC payments from resulting congestion revenues did not necessarily indicate market inefficiency.

In their follow-up work (Siddiqui et al. 2006), they re-analyzed the New York TCC data, taking into account possible risk aversion of market participants. Instead of using a linear model, they employed three different concave "utility functions" and fitted nonlinear regressions to the TCC payments and revenues data[25]. Their results showed that market participants were only slightly risk averse (or even risk seeking, depending on the utility function employed). Thus, risk aversion by itself could not fully explain the systematic divergence between the TCC prices and congestion rents. The authors concluded that it was the very design of these markets, rather than the behavior of market participants that led to the observed discrepancy between prices and revenues.

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[24] TCC: Transmission Congestion Contract, is one implemented form of FTR.

[25] We put question marks on the way Siddiqui et al. (2006) use "utility" functions in their regression specifications in that the direct substitution of the FTR prices in the form of concave utility function is not justified.

In spite of their original empirical work in evaluating the FTR auction market, we would like to point out a somewhat subtle and yet critical problem in Siddiqui et al. (2005) and (2006). That is, in these two papers, they treat the ex-post realized data as if they were ex-ante to match their empirical methods without stating clearly the underlying assumptions and their justifications for doing so. Given the situation that most of the ex-ante data are difficult or even impossible to obtain (confidentiality issue for example), one contribution of our paper is to try to state clearly the underlying assumptions we make to bridge the inconsistency between ex-ante method and ex-post data, and provide some theoretical foundations to the empirical methods we use.

In our study, we focus on the FTR market in the Midwest energy region (MISO), which has never been investigated empirically in the literature. Compared with the other FTR markets such as New York's, the MISO FTR market has a much shorter history. The scarcity of available data with MISO poses a great challenge to reaching any complete conclusion about this emerging market. Unavoidably, we need to make assumptions in order to analyze, to the best extent, the data we could obtain. Although some of the assumptions cannot be tested for the moment due to insufficient data, we will be able to check for validity of these assumptions once more data become available. Through our analysis, we find a number of stylized facts as well as evidence of the performance and efficiency of the MISO FTR market. In addition, we make suggestions about improving data collections by MISO or other research entities, so that more in-depth studies could be carried out in future to evaluate and monitor this huge market. Our work is original in that no one has ever conducted any empirical study on this new market. Besides, our work is not restricted to the partial analysis due to limited data, but sheds light on what needs to be done to make the data more complete and meaningful.

The rest of the chapter is organized as follows. Section 4.2 provides some background information about the MISO energy and FTR auction markets. In section 4.3, we review the underlying theory of hedging and risk preferences on which our analysis is based. Section 4.4 provides a detailed description of the data. The empirical methods employed and assumptions are discussed in section 4.5. In section 4.6, we present and interpret our results. Finally, the concluding remarks are given in section 4.7.

4.2 MISO Energy and FTR Markets

Founded on February 12, 1996, MISO is an independent and non-profit organization whose primary roles are to provide equal access to the transmission system and ensure reliable and efficient electric system in a competitive wholesale power market in the Midwest region. Currently MISO is managing transmission operations for all or part of 15 U.S. states plus Manitoba province in Canada.

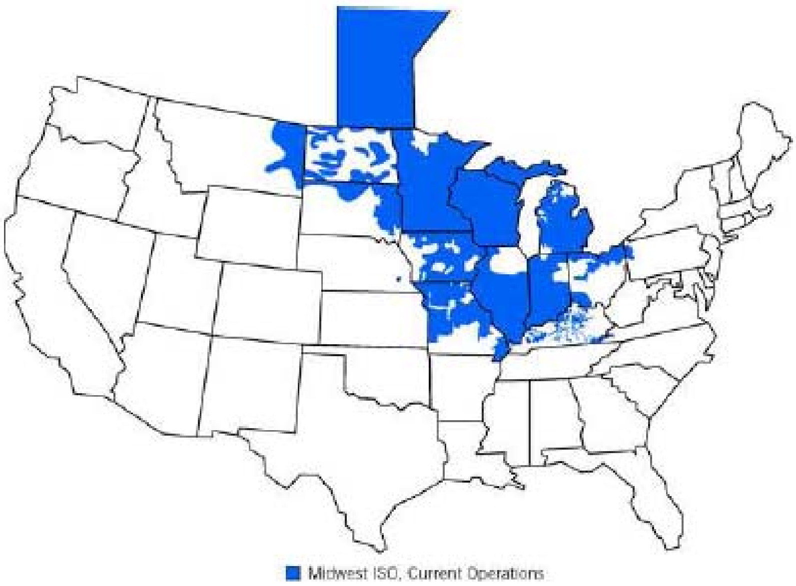

Figure 1: The current MISO service territory

(Source: MISO, http://www.midwestiso.org/page/About%20MISO)

Figure 1 shows the current MISO operation territory. Since April 2005, MISO has been operating a day-ahead energy market, a real-time energy market and an FTR market. The day-ahead market is a forward market in which hourly LMPs are calculated for each hour of the next operating day. According to MISO's Market Concepts Study Guide (MISO 2005c), the day-ahead market is cleared using the security-constrained unit commitment (SCUC) and security-constrained economic dispatch (SCED) algorithms to satisfy energy demand bid and supply offer requirements. To be specific, the objective in clearing the day-ahead market is to minimize the costs of day-ahead energy procurement over the 24-hour dispatch horizon based on the offers and bids, subject to network constraints and resource operating constraints. The results of the day-ahead market clearing include hourly LMP values and hourly demand and supply quantities, which are posted on MISO's market portal on 1700 hours EST. The real-time energy market, in contrast, is an instant balancing market in which the LMPs are calculated every five minutes, based on MISO dispatch instructions and actual system operations. These two markets operate in a coordinated sequence and are settled separately. In the settlement of the day-ahead market each hourly MW injection is paid the day-ahead LMP at its node and withdrawals are charged the day-ahead LMP at their respective nodes. The day-ahead LMPs are also used to establish the settlement value of FTRs and bilateral transactions. The real-time settlement is based on actual hourly quantity deviations from the day-ahead scheduled quantities and on real-time prices integrated over the hour. Any deviation in the quantity from the day-ahead schedule (including bilateral transactions) is charged or paid real-time LMPs.

4.2.1 LMP Components

Since FTRs crucially depend on locational marginal prices (LMPs), it is important to take a close examination on the LMP and its components. By definition, LMP at any given pricing location is the minimum incremental cost of servicing one additional unit of demand at that location under the constraints of production, congestion and transmission losses. LMPs vary by time and location. Variability of LMPs is due to the physical constraints, congestion and losses. For each node, MISO determines three separate components of its LMP, namely the marginal energy component (MEC), marginal congestion component (MCC) and marginal loss component (MLC). MEC is the LMP of the reference node, so is the same for all the nodes. MCC and MLC of a certain node represent the marginal cost of congestion and marginal cost of losses, respectively at that node relative to the reference node.

$$LMP_n = MEC_r + MCC_n + MLC_n$$

$$LMP_r = MEC_r$$

where *r* is the reference node and *n* is any node other than the reference one.

Of the three LMP components, MEC_r is calculated as the marginal cost of energy at the reference node *r*, so is determined by the cost functions of the generators at that node. The congestion component MCC_n is calculated as follows:

$$MCC_n = -(\sum_{k=1}^{K} GSF_{nk} \times FSP_k)$$

where K is the number of thermal or interface transmission constraints (also called *flowgates*), GSF_{nk} is the shift or distribution factor for the generation at node n on flowgate k and FSP_k is the shadow price of the thermal limit on flowgate k. Intuitively, GSF_{nk} is the proportion of each MW injected at node n and withdrawn at the reference node r, and FSP_k is the cost saved from one MW increase in the capacity of flowgate k[26]. In the Midwest market, congestion is handled financially through the MCC of the LMP, and the congestion revenue from holding the FTR is determined by the difference in MCCs.

MLC_n is calculated using the equation

$$MLC_n = (DF_n - 1) \times MEC_r$$

where DF_n is the delivery factor for node n to the reference node. DF_n is equal to $1 - \frac{\partial L}{\partial G_n}$, where L is system losses and G_n is the amount of power injected at node n. Therefore, $\frac{\partial L}{\partial G_n}$ is the change in system losses due to an incremental change in the power injection at node n holding everything else constant.

4.2.2 Overview of MISO FTR Acquisition

FTRs are tradable financial instruments that allow market participants to hedge against the cost and uncertainty that may arise from congestion in the transmission grid. The FTR holders are entitled to a stream of revenues or charges based on the congestion over the FTR path. FTRs are used in the day-ahead market only and do not apply to the real-time market. They do not protect market participants from congestion charges related to scheduling power in the real-time market or deviating from the day-ahead schedule. Nor do they hedge against transmission loss charges. Besides, FTRs are independent of the physical power dispatch. The FTR holder has the financial right to the congestion between two specified nodes regardless of the actual energy deliveries.

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[26] According to the industry convention, the effect of losses is ignored in determining *GSFs*.

An FTR is specified by its source and sink, the MW amount, the term for which the FTR is in effect, the time period (peak or off-peak hours), and whether the FTR is an obligation or option. FTR options are currently not available in the MISO market. An FTR obligation grants the holder the right to collect, for each MW of FTR, the congestion rent accumulated from the source to the sink for every hour during the effective period. The congestion rent is determined by the difference between the congestion components in the day-head LMPs at the sink and source. Therefore, an FTR obligation can have a positive or negative economic value, depending on the actual congestion pattern between the source and sink on which the FTR is defined. During the hours when the congestion component at the sink is greater than the congestion component at the source, the FTR yields a positive revenue to the holder. If, instead, the congestion occurs from the sink to the source, the holder of the FTR will have to pay MISO an amount equal to the congestion rent in the congested direction, or equivalently receive a negative revenue.

In the Midwest, market participants can acquire FTRs through allocations (annual and monthly), auctions (annual and monthly) and the secondary market. FTRs are first allocated in the annual allocation based on existing entitlements from transmission service reservations and grandfathered agreements. The annual FTR auction is held right after the annual allocation and prior to the beginning of each year for the subsequent four seasons[27]. In this auction, market participants can submit offers to sell or bids to buy FTRs and MISO determines the winning sellers and buyers. In order to be eligible for the annual auction, the FTR must be valid for the entire period of the seasons in the auction. A monthly allocation is performed for each operating month to come. Those FTRs eligible in the initial allocation that did not receive their full entitlement in FTR awards can be re-considered in this monthly allocation process. Then after the monthly allocation takes place, the monthly auction is conducted. Any FTR eligible for the monthly auction must be valid for the entire month in the auction. The exact timeline for the MISO monthly FTR allocations and auctions are given in Appendix A for a sample month (August 2005). There is also a secondary market for buying and selling FTRs. The FTR allocations are irrelevant to our research purpose in this

[27] The four seasons are: (i) Winter: December, January, February; (ii) Spring: March, April, May; (iii) Summer: June, July, August; (iv) Fall: September, October, November.

paper, as there is no market or price for the allocation. Therefore, we do not consider the allocations in evaluating the market performance. In both the annual and monthly auctions, FTRs are sold and bought at the market clearing prices, the determination of which will be elaborated later. However, the data for the annual allocations are not sufficient since MISO has only adopted FERC's WPMP design since March 2005. In addition, a time interval of three months may be too long for discerning any change or trend in this market during the one-year period. Therefore, a monthly basis is proper for our research purpose. Although the secondary market is also relevant to our study, we have to ignore it, because little information is available about it. In all, considering relevance and availability, we finally choose to focus on the monthly FTR auctions and ignore the possible effects of other means to obtain FTRs.

4.2.3 MISO Monthly FTR Auctions

MISO conducts monthly FTR auctions for two purposes: (1) to allow MISO to sell FTRs for the adjusted monthly FTR capability of the market footprint, and (2) to facilitate the buying and selling of existing FTRs between market participants. Market participants buy from or sell to the available "pool" of system FTR capacity. All FTRs at the monthly auction have a term of one month beginning on the first day of the month following the auction. Market participants must submit their offers or bids to MISO during the monthly bidding period and MISO posts the auction results no later than 5 business days before the start of the subject month (see Appendix A for a more detailed MISO FTR allocation and auction timeline for a sample month - August 2005). Each monthly auction consists of two separate auctions: one for the peak period and the other for the off-peak period. Peak is the period of time ending 0600 hours Eastern Standard Time (EST) to 2200 hours EST on weekdays excluding holidays[28]. Off-peak is all periods not classified as peak. The purchaser of a peak (off-peak) FTR from the monthly auction is entitled to the aggregate congestion rents of the peak (off-peak) hours during the whole month.

FTR bidders are responsible for submitting a bid that indicates the following:

1. Type of FTR (obligation or option)

[28] These holidays are specified by North American Electric Reliability Council (NERC).

2. FTR source and sink

3. Maximum MW quantity desired

4. Maximum acceptable price, in $/MW

5. Period (peak or off-peak)

Similarly, FTR sellers should submit an offer including the above items 1 through 5 except that item 3 now becomes the maximum MW quantity offered instead of desired. Given the offers and bids for each monthly auction, MISO determines the winners (traders that get cleared, i.e., the infra-marginal traders) as well as FTR clearing quantities and clearing prices by solving a linear programming problem. Specifically, it maximizes the value of FTRs bought minus FTRs sold by auction participants subject to simultaneous feasibility constraints with "n-1" security constraints. All comparable FTRs are sold at the same market clearing price expressed in $/MW, which is calculated as the difference in the shadow price of the power flow balance constraint at the FTR source and sink in the FTR auction linear programming problem above. It can be interpreted as the negative of the marginal change in the objective function value due to an infinitesimal change in the flow from the FTR source to the sink. It is worth noting that the FTR clearing price can be negative, which means that the market participants who buy the FTR will receive money from MISO whereas those who sell the FTR will pay money to MISO. This usually happens when the particular line associated with this FTR is anticipated to be congested in the direction opposite to that specified by the FTR.

4.3 Theory

In this section, we first illustrate, through several examples, the role of FTRs in hedging against risks caused by the volatile location marginal prices (LMPs). In doing so, we consider two cases: electricity transaction scheduled on bilateral agreements and electricity transaction purely via the marked-based power pool. Then we demonstrate the relationship between the expected revenue from holding FTRs and the agent's willingness to pay to get them for both risk-neutral and risk-averse agents.

4.3.1 Hedging Role of FTRs

LMPs in the electricity wholesale market provide the right incentives for generation and consumption, but also create a need to hedge the price changes. This leads to the interest in FTRs. Electricity transactions are usually settled through bilateral schedules or via the market-based power pool. In either case, FTRs provide a hedge against the congestion charge by reimbursing the holders part or all of the charge. How FTR works as a hedging instrument is illustrated through following several examples[29].

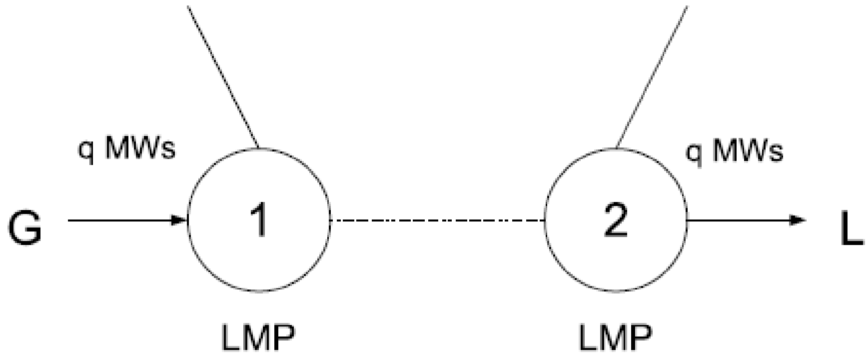

Figure 2: Bilateral contract with no congestion

Let us first consider the case of bilateral agreements. If there were no transmission congestion, it is well justified to treat all production and consumption as if they took place in the same location since both buyer and seller are settled with the same price, which is the single equilibrium price in the market. Then the natural arrangement is to contract for differences against the equilibrium price. As depicted in Figure 2, a GENCO (G) and an LSE (L) are attached to node 1 and node 2, respectively. The two nodes do not have to be directly connected by a transmission line, so we use the dashed between them. The nodes and lines can be only part of a larger network, which is not drawn in the figure. Suppose G and L agree on a price of p_B (\$/MWh) for trading a fixed quantity of electricity q MWs at a specific hour. If there is no congestion in the network in this hour, the prices (or more precisely LMPs) at all nodes would turn out to be the same, hence denoted by a common price *LMP*. G will then sell electricity to the market at *LMP* and L will buy electricity from the market at *LMP*. Note that the q MWs that L purchases do not have to be produced by G, and the electricity that G

[29] For simplicity, we do not consider transmission losses in our illustrative examples.

generates may be bought by other LSEs. So the arrows indicate the direction of the contract path. If $LMP > p_B$, under the contract, G owes L $LMP - p_B$ for each of the q MWs over this hour. In the opposite, if $p_B > LMP$, L owes G $p_B - LMP$ for each of the q MWs over this hour. This is the so-called *contract for difference* (CFD), which locks the actual transaction price at p_B for both G and L and provides a perfect hedge against the price risk. Therefore, in the absence of congestion, a bilateral arrangement between the GENCO and the LSE can capture the effect of aggregate movements in the market, as the single market price gets up or down over time.

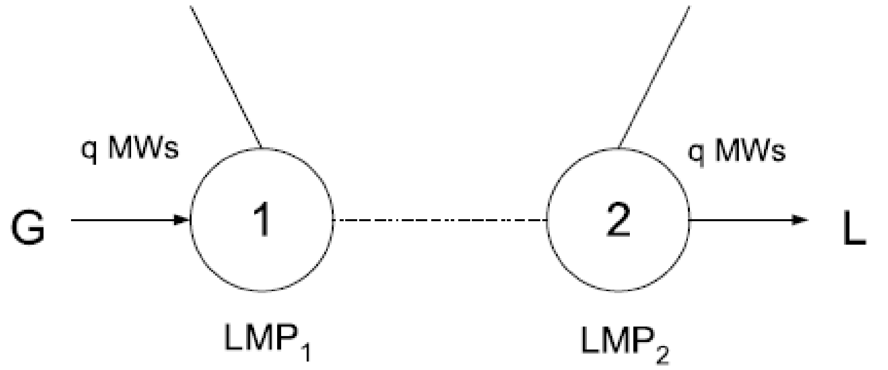

Figure 3: Bilateral contract with congestion

Most of the time, however, there is congestion somewhere in the transmission network. In that case, the price will differ depending on the location and G and L may no longer face the same price. This situation is illustrated in Figure 3. Let LMP_1 and LMP_2 be the locational marginal price at node 1 and 2, respectively and $LMP_1 \neq LMP_2$ due to the congestion. Also let p_B be the bilaterally agreed contract price between G and L. Then at the settlement G will sell electricity to the market for LMP_1 and L will buy electricity from the market for LMP_2. If $LMP_1 < p_B < LMP_2$, then L pays $LMP_2 - p_B$ more than the contracted price and needs to be compensated for the excessive payment. On the other hand, G receives $p_B - LMP_1$ less than the contracted price, so also needs to be compensated. Obviously, it is impossible to satisfy both parties only through the CFD and something else is needed to complement the CDF.

Still consider the situation in Figure 3. Now suppose that apart from the CFD, G may obtain an FTR of q MWs defined from node 1 to node 2. The FTR entitles G to the difference in the LMPs between the two nodes, i.e. $LMP_2 - LMP_1$ for that hour. L pays LMP_2 for the power. The settlement system in turn pays G LMP_1 for the power supplied. Thanks to the FTR, G ends up receiving LMP_2 for each MWs sold, and L ends up paying LMP_2 for each MW bought. In this sense, we come back to the previous situation in which the price is the same at all nodes, and the CFD will work out now. If $LMP_2 > p_B$, G will compensate L for its excessive payment $LMP_2 - p_B$ according to the CFD. If $LMP_2 < p_B$, G will be compensated for the loss $p_B - LMP_2$ by L. As a result, no matter how the LMPs change, the transaction would be effectively settled at the deterministic, bilaterally agreed price p_B. On the other hand, L can also buy the same FTR and the result will be the same. The example indicates that in the presence of transmission congestion, an FTR together with a CFD can provide full hedge against the risk associated with the LMPs. The function of FTRs in the scenario of bilateral contract actually lies in equalizing the price that the selling and buying parties face, which provides the condition for the CFD to be workable. The FTR provides hedge against locational price risks, while the CFD, against temporal price risks. In this example, the FTR together with CFD provides a perfect hedge for both parties, because the quantity of FTRs obtained exactly matches the contracted quantity of electricity. If G or L buys the FTR for an amount less than the contracted quantity of electricity, they will only have partial coverage.

In summary, a seller and a buyer entering into a bilateral transaction of electricity between two nodes can always hedge against the price risk by using the FTR and CFD jointly.

Now let us come to the case with no bilateral schedules. That is, each GENCO simply injects electricity to the power pool and gets the LMP at its own node for each MW injected. Each LSE withdraws electricity from the same power pool and pays the LMP at its own node for each MW withdrawn. Hence a GENCO does not know or care about where its power is withdrawn and who actually buys the power it has produced. Similarly, an LSE does not know where the power it buys comes from and who generates it. This is different from the transaction under a bilateral contract in which the seller and the buyer as well as the injection and ejection nodes are designated and the transaction price is predetermined[30]. In the bilateral contract case, a combination of the FTR and CFD can provide a perfect hedge and make the payment and revenue nonrandom. Without a bilateral contract, however, it is not obvious to a market participant between which two nodes she pays the congestion charge. Hence she does not know for sure between which two nodes she should obtain the FTR to hedge the potential congestion. We can see this from the following example as depicted in Figure 4.

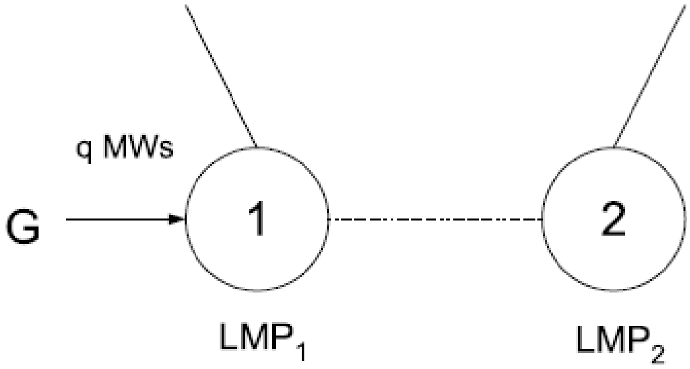

Figure 4: Transaction via a power pool with congestion

This figure looks similar to Figure 3, but notice that only the q MWs injection of G is given. G does not know where these q MWs would be withdrawn or through what paths they would be transited. G only knows that it would get LMP_1 for each MW it produces. LMP_1 is volatile and fluctuates with the generation, loads and congestion patterns. G is exposed to the price risk and might want to hedge against it. Suppose that LMP_2, for some reason, is relatively stable and does not change much[31]. If G purchases an FTR of q MWs from node 1 to node 2, it would get $LMP_1 + (LMP_2 - LMP_1) = LMP_2$ for each MW generated. In this sense, holding the FTR reduces G's price uncertainty by making its revenue less volatile. If LMP_2 is virtually nonrandom, then the FTR provides a perfect hedge for G. If LMP_2 is also random, but is much more stable than LMP_1, then the FTR provides a non-perfect partial hedge. To reduce its revenue uncertainty, G could effectively associate its revenue with LMP_2 by holding the FTR from node 1 to node 2.

[30] Though in the case of bilateral transaction, the q MWs purchased by the LSE may not be the same q MWs that GENCO produces, either.
[31] This might occur at a hub which consists of a set of nodes and whose LMP is derived from the average of the LMPs of those nodes.

This is not the only choice for the hedging. In fact, G can choose a set of FTRs (portfolio of FTRs) to achieve the same result. Let us take another example with a three-node network in Figure 5.

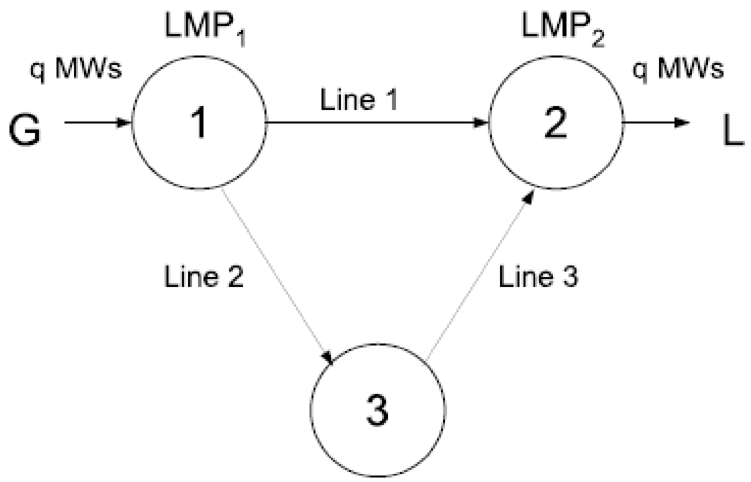

Figure 5: The three-node electric network example

In this example, the only generator G is located at node 1 and the only LSE L is located at node 2. For computational simplicity, assume that each line has the same impedance. Then of each MW injected at node 1, $\frac{2}{3}$, $\frac{1}{3}$ and $\frac{1}{3}$ will move along line 1, line 2 and line 3, respectively. Clearly, G will have a revenue of $q \times LMP_1$ from injecting q MWs of electricity at node 1. To hedge against the congestion charge, G may obtain q MWs of FTR defined from node 1 to node 2. Then G's total revenue will be $q \times LMP_1 + q \times (LMP_2 - LMP_1) = qLMP_2$. As mentioned earlier, if LMP_2 is nonrandom, then the FTR provides G with a perfect hedge. This is not the only way to achieve the deterministic revenue $qLMP_2$. An alternative could be that G obtains a portfolio of FTRs with $\frac{2}{3}q$ MWs of FTR from node 1 to node 2, $\frac{1}{3}q$ MWs of FTRs from node 1 to node 3 and $\frac{1}{3}q$ MWs of FTRs from node 3 to node 2. G will again end up with $q \times LMP_1 + \frac{2}{3}q \times (LMP_2 - LMP_1) + \frac{1}{3}q(LMP_3 - LMP_1) + \frac{1}{3}q(LMP_2 - LMP_3) = qLMP_2$. This FTR portfolio also links G's revenue to LMP_2, which is less volatile than LMP_1. As will be seen later, market participants in practice sometimes purchase more than one single FTR and indeed construct FTR portfolios.

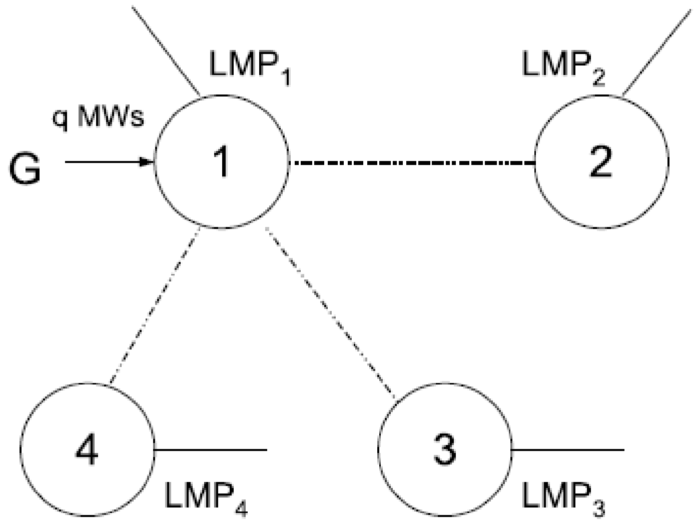

Figure 6: Transaction via a pool with congestion: FTR portfolio

Figure 6 gives another example of hedging using an FTR portfolio. Similar to the previous examples, G injects q MWs to the power pool and receive payments at its own nodal price LMP_1 for each MW injected. Since LMP_1 fluctuates, G may want to hedge against the uncertainty using FTRs. Suppose that the LMPs at nodes 2, 3 and 4 change almost independently, such that the pairwise correlations between the LMPs at the nodes 2, 3 and 4 are very small. Let σ_n^2 be the variance of the LMP at node n, for $n = 1,2,3,4$. Then G can reduce the price risk by holding a portfolio of FTRs associated with the nodes 2, 3 and 4. Let $q_{12}, q_{13}, q_{14} \geq 0$ be the quantity of MWs that G obtains for the FTR from 1 to 2, from 1 to 3 and from 1 to 4, respectively and assume that $q_{12} + q_{13} + q_{14} = q$. Consequently, G will get

$$\begin{aligned} &LMP_1 q + (LMP_2 - LMP_1)q_{12} + (LMP_3 - LMP_1)q_{13} + (LMP_4 - LMP_1)q_{14} \\ &= q_{12}LMP_2 + q_{13}LMP_3 + q_{14}LMP_4 \end{aligned}$$

which is a weighted average of the LMPs of the three nodes. This revenue is still random and its variance can be calculated as

$$\begin{aligned}&\text{var}(q_{12}LMP_2 + q_{13}LMP_3 + q_{14}LMP_4)\\&= q_{12}^2\sigma_2^2 + q_{13}^2\sigma_3^2 + q_{14}^2\sigma_4^2 + 2q_{12}q_{13}\sigma_{23} + 2q_{12}q_{14}\sigma_{24} + 2q_{13}q_{14}\sigma_{34}\\&= q_{12}^2\sigma_2^2 + q_{13}^2\sigma_3^2 + q_{14}^2\sigma_4^2\end{aligned}$$

where σ_{23}, σ_{24} and σ_{34} are the pairwise covariances of the LMPs at nodes 2, 3 and 4. The second equality holds when the three LMPs are pairwisely uncorrelated. The variance of G's revenue in the absence of the FTR portfolio is

$$\text{var}(qLMP_1) = (q_{12} + q_{13} + q_{14})^2\sigma_1^2$$

If LMP_2, LMP_3 and LMP_4 are almost constant, that is, σ_2^2, σ_3^2 and σ_4^2 are nearly zero, then $\text{var}(q_{12}LMP_2 + q_{13}LMP_3 + q_{14}LMP_4) \approx 0$ and holding the FTRs completely eliminates the risk associated with LMP_1. If LMP_2, LMP_3 and LMP_4 are no more volatile than LMP_1, that is, if $\sigma_2^2 \le \sigma_1^2$, $\sigma_3^2 \le \sigma_1^2$ and $\sigma_4^2 \le \sigma_1^2$, then

$$\text{var}(q_{12}LMP_2 + q_{13}LMP_3 + q_{14}LMP_4) \le \text{var}(qLMP_1)$$

This relationship may also hold if LMP_2, LMP_3 and LMP_4 are negatively correlated. Hence, holding this FTR portfolio lowers G's risk exposure, although the portfolio may not completely eliminate the risk. In all, the FTRs provide G with a hedge, but not necessarily a perfect one.

From the above examples, we find that under a bilateral agreement, FTRs together with CFDs can always provide a perfect hedge against the price risk and a GENCO ends up with a fixed revenue. In comparison, in the absence of bilateral agreements, the resulting revenue that a GENCO receives from selling electricity and holding FTRs is usually a function of the LMPs. So the revenue may not be fixed and its variability depends on the variations of LMPs and their covariances. Under some conditions, this variability of G's revenue is less than the one if G does not have FTRs, and FTRs provide a hedge against the risk. The hedging functionality of FTRs is similar for LSEs who purchase, rather than sell power. In analogy, holding FTRs can also provide a hedge against the volatile revenue of an LSE.

A complete analysis of the hedging or coverage provided by FTRs requires knowledge of the power transactions that market participants enter into -- whether it is through bilateral contract or via the power pool. The literature, however, often ignores the type of power transactions which motivates the market participants to obtain FTRs and only focuses on the random congestion charges. Knowing that the volatility in congestion charges comes from the volatility in LMPs which the transactions are based on, most papers conclude that a market participant simply has to purchase enough FTRs to hedge its transmission congestion exposure perfectly. They regard the congestion charges as the only random variables and make their reasoning in this way: since market participants can have these charges reimbursed by holding FTRs, FTRs will provide a perfect hedge. This, as can be seen from our previous examples, generally does not hold. An agent chooses FTRs to hedge against the risk with the LMP exposure, not simply the congestion charges. The FTRs may provide a full coverage for the congestion payment but not for the market participant's total income from power transactions via the pool.

4.3.2 Theoretic Framework

In this subsection, we explore the relationship between the expected congestion charge or equivalently the expected revenue from holding an FTR and the agent's willingness to pay for that FTR. Two types of risk preference are considered -- risk neutrality and risk aversion. For risk-neutral case, we can derive the relationship between the revenue and the payment analytically. However, it is much harder to do so for risk-averse case. Alternatively, instead of providing a rigorous proof as we did for the risk-neutral case, we will try to identify the relationship between the expected congestion revenue and the FTR payment through numerical simulation.

Although from the previous subsection, we know that holding FTRs may not be able to eliminate completely the uncertainty with an agent's income in some cases, here we make assumption that FTRs will always provide a perfect hedge for our theoretical analysis. That is, an agent can make her profit deterministic or free of price risk by holding an FTR. Let w denote the non-stochastic total profit of an agent resulting from obtaining an FTR. Then without the FTR, the agent's profit is $w-R$, where R is the congestion charge (or revenue) the agent has to pay and is a random variable. Hence, $w-R$ is also a random variable and the agent's profit is uncertain. An FTR reimburses the congestion charge R for the agent no matter how much R is. So the expected revenue from holding the FTR is $E(R)$. As a result, owning the FTR locks the agent's profit at w and provides a perfect hedge against the risk. Of course the agent has to pay in order to acquire the FTR. We will use F to denote the maximum \$ amount she is willing to pay for the FTR.

Consider a risk-neutral agent whose utility function can be expressed as

$$U(\pi) = a + b\pi$$

where $b > 0$ and π is the agent's profit or payoff. Suppose that the congestion charge R is distributed according to a probability density function (PDF) $f(R)$. Then in the absence of FTRs, the agent's profit $\pi = w - R$ is a random variable and her expected utility is given by

$$\begin{aligned} EU &= \int U(w - R) f(R) dR \\ &= a + b(w - ER) \end{aligned}$$

If she purchases FTR for \$$F$, her utility will be

$$U(w - F) = a + b(w - F)$$

By certainty equivalence theory, the agent's willingness to pay (F) for the FTR satisfies

$$EU = U(w - F)$$

Hence,

$$F = E(R) \tag{1}$$

which means that a risk-neutral agent is willing to pay up to exactly the expected charge/revenue from holding the FTR. Note that the relationship still holds if we divide both sides of equation (1) by any positive constant. Choosing the MWs of the FTR as the divisor, we then have that the expected congestion revenue of one MW FTR or the expected unit revenue (which is equal to the sink LMP less the source LMP) should be equal to the unit willingness to pay under risk neutrality. This justifies our use of congestion revenue per MW of FTR and the unit price in later analysis, instead of multiplying them by the quantity purchased. So if the expected unit revenue is plotted against the unit willingness to pay, it should be a 45 degree line if all agents are risk neutral.

Now consider a risk-averse agent who would be willing to pay extra premium to stabilize his profit to a deterministic amount. By certainty equivalence theory, we have the following equation:

$$EU(w-R)=U(w-F) \quad (2)$$

By Jensen's inequality, it follows that

$$U(w-F)=EU(w-R)<U(E(w-R))=U(w-ER) \quad (3)$$

Then it is straightforward that the willingness-to-pay for FTR is always greater than the expected value for congestion charge. That is,

$$F > ER \quad (4)$$

This means that a risk-averse agent is willing to pay more than the expected charge or revenue from holding the FTR. However, the further relationship between F and ER depends not only on the entire distribution of random variable R but also on the utility function $U(\cdot)$. It is beyond the scope of this paper to explore such relationship in full detail.

4.4 Data

Our study focuses on the one-month FTRs that were purchased in the monthly FTR auctions in MISO from April 2005 to May 2006. The data we use are obtained from the FTR auction results of the twelve months and the historical day-ahead LMP files, which are all publicly available in MISO's website. Specifically, the results of each monthly auction include, for each FTR, the buyer, the source and sink, the MW amount awarded, the class (peak or off-peak), whether it is an obligation or option and the market clearing price measured in \$/MW[32]. For each MW of FTR awarded, the buyer must pay the clearing price of that FTR, which is determined in the auction for each month t. So the clearing price is actually the unit cost of obtaining an FTR. Let $F_t^{m,n}$ denote the market clearing price of the FTR defined from node m to node n for month t, $t=1,...,T$.

The auction results do not directly report the unit revenue from holding an FTR, that is, the congestion rent per MW accumulated from the source to the sink for the effective month. Hence in the data pre-processing stage, we calculate the unit revenue using the congestion component (MCC) of the day-ahead LMPs, which can be found in the historical day-ahead LMP files. For each of the twelve sample months, the historical day-ahead LMP dataset includes the MCC of each node for each of the 24 hours in each day of that month.

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[32] So far, all the FTRs auctioned in MISO are obligations.

For example, in April 2005, there are 30 days, each having 24 hours. So, for a node such as WPS.PULLIAM3, we have 24×30=720 hourly MCCs. The frame of the hourly MCCs for node WPS.PULLIAM3 in April 2005 is given in Table 1 as follows.

Table 1: Hourly MCCs of node WPS.PULLIAM3 in May 2005

MCC	$d=1$	$d=2$	$d=3$	...	$d=31$
$h=1$	0.04	0.46	2.89	...	0.62
$h=2$	0.02	0.88	2.99	...	1.69
$h=3$	0.04	0.60	2.92	...	1.67
...	...	...	...	...	...
$h=24$	0.04	0.29	1.34	...	3.79

The value in each cell in the figure is the MCC of node WPS.PULLIAM3 for the corresponding hour h of the corresponding day d in May 2005. For example, the cell corresponding to $h=1$ and $d=1$ is the MCC of node WPS.PULLIAM3 in the first hour on May 1, 2005. This table structure applies to all the other nodes in each month of the sample period. During the twelve months in our study, the nodes and number of nodes for each month stayed the same within that month, but might not, across months. As time went on, new nodes were added to MISO's transmission network or some existing nodes were removed from it. The number of nodes for each month as well as the change in the number from the previous to the current month is given in Table 2.

Given the MCC data described above, the revenue that the FTR owner gets for each MW of FTR held can be derived as the sum of the difference between the hourly sink and source MCCs in the day-ahead market over all hours in the effective month. Let $h=1,...,24$ index the hour, and $d=1,...,D_t$ index the day, where D_t is the number of days in month *t*. Let p^n_{hdt} be the MCC of node *n* at hour *h* on day *d* in month *t*. Then $R^{m,n}_t$, the revenue from holding one MW of the FTR from node m to node n during month t can be computed as

$$R^{m,n}_t = \sum_d \sum_h \left(p^n_{hdt} - p^m_{hdt}\right) \qquad (6)$$

We can also call $R^{m,n}_t$ the unit revenue of the FTR from *m* to *n*. Note that for peak FTRs, the revenues are calculated by aggregating MCC differences in the peak hours; for off-peak

FTRs, the MCC differences used in calculating the revenues are all those in the off-peak hours. So, in equation (6), the range of d and h over which the MCC difference is aggregated depends on whether the FTR is for peak or off-peak hours.

Table 2: Reported number of nodes in MISO service region (April 2005-March 2006)

Month	Number of nodes	Change	Percentage
Apr-05	1471		
May-05	1471	0	0.00%
Jun-05	1496	25	1.70%
Jul-05	1496	0	0.00%
Aug-05	1496	0	0.00%
Sep-05	1508	12	0.80%
Oct-05	1508	0	0.00%
Nov-05	1508	0	0.00%
Dec-05	1535	27	1.79%
Jan-06	1536	1	0.07%
Feb-06	1536	0	0.00%
Mar-06	1520	-16	-1.04%

We notice that in some months, some sources or sinks on which the FTRs were defined cannot be found in the list of the day-ahead LMP file of the corresponding months. In that case, we are unable to calculate the revenue from holding those FTRs. For example, in June 2005, the market participant EMMT bought 5.8 MWs of the FTR from node NSP.CHARA6 to node GRE.WILM, but the source node is not found in the LMP file for the same month. Such discrepancies occur because the commercial model changed after the FTR auction results were finalized. For example, the June 2005 auction was conducted in May 2005. The June commercial model was propagated into the FTR system after the June 2005 auction was completed. Sources and sinks on the awarded FTRs were corrected to match the updated model, but the auction result report was not updated since it reflected the actual outcome from the auction as it was conducted. A snapshot of the most current active FTRs in the system is available on the portal, but not currently posted on MISO public website. In our analysis, we shall ignore any FTR defined on the "missing" nodes.

Table 3: Reported number of distinct FTRs (April 2005-March 2006)

Month	Number of distinct FTRs	Month	Number of distinct FTRs
Apr-05	164	Oct-05	932
May-05	255	Nov-05	1448
Jun-05	349	Dec-05	1446
Jul-05	863	Jan-06	1278
Aug-05	989	Feb-06	2105
Sep-05	870	Mar-06	1761

Similar to the number of nodes, the number of FTRs purchased in the monthly auctions was not the same for each month either. More FTRs were bought in some months than others, as is shown in Table 3. Several factors contribute to the fluctuating number of the FTRs purchased. As the number of nodes on which FTRs are defined varies across month, the change in the FTR purchase number is natural and understandable. For example, if new nodes are added to the network, more FTRs will be available, since they can be defined on more combinations of nodes. Besides, seasonality may also cause more FTRs to be purchased in some months than others. With only one year's data, we cannot tell much about the effect of seasonality on the FTRs purchased. A longer sample period for a relatively stable transmission network is needed for analyzing the seasonality effect. This will be possible for MISO as time goes on.

Before proceeding, let us introduce some notations for FTR types that is applied systematically throughout the rest of this paper. Since all auctioned FTR can be classified as either peak or off-peak, and people's purchasing decisions regarding peak and off-peak FTRs are expected to be different, we classify our data to two parts: peak and off-peak accordingly[33]. In addition, since there are certain FTRs (defined by point of source and point of sink) that are purchased by more than one buyer, and since all of them will receive the same prices and congestion rents, we will also distinguish our data between non-distinct and

[33] According to MISO (2005a), peak periods are defined as: weekdays, for hours ending 0700 to 2200 hours Eastern Standard Time (EST), and excluding North American Electric Reliability Council (NERC) holidays. Off-peak periods are defined as: weekdays, for hours ending 0100 to 0600 hours EST and hours ending 2300 to 2400 EST, weekends, and NERC holidays, for all hours.

distinct. The non-distinct data are simply the original data while the distinct data are the data where duplicated FTR purchases on the same FTRs are removed.

By this POND (Peak, Off-peak, Non-distinct, Distinct) classification, we will have four different classes of FTRs over the period April 2005 - March 2006. These four classes of FTRs are: ON (Off-peak and Non-distinct), OD (Off-peak and Distinct), PN (Peak and Non-distinct) and PD (Peak and Distinct). All the statistical computations are implemented using R[34].

A quick overview of the data gives us some stylized facts of the MISO's FTR market. First of all, for all four types of FTRs, the average clearing prices and congestion revenues collected are highly volatile across all months. For example, the average clearing price of PD FTRs in April 2005 is highly negative (-6313.36), while the value in January 2006 is highly positive (960.31). This might be a sign of an immature and unstable new market. The second observation is that for most of the FTRs awarded, there was only one buyer for each FTR. In other words, the difference between distinct and non-distinct FTRs is not large, which may imply that the MISO monthly FTR auction market is quite thin. See Table 4 for reported number of non-distinct and distinct FTRs and their difference for the sample period.

The result from Table 4 indicates that the liquidity of the monthly MISO FTR market increases during the one-year period. In April 2005, the market was so illiquid that there were only 3 more non-distinct FTRs than distinct FTRs. The overall tendency in the difference was increasing as time went on, although the difference in June, September 2005 and March 2006 decreased from the previous month. These fluctuations may be explained by seasonality factors, but we need more information to provide a definite answer.

[34] R is an open-source software environment for statistical computing and graphics. More information about R can be found at http://www.r-project.org.

Table 4: Reported number of non-distinct and distinct FTRs (April 2005-March 2006)

Month	NumN[a]	NumD[b]	NumN-NumD
Apr-05	167	164	3
May-05	311	255	56
Jun-05	353	349	4
Jul-05	890	863	27
Aug-05	1062	989	73
Sep-05	911	870	41
Oct-05	1011	932	79
Nov-05	1569	1448	121
Dec-05	1707	1446	261
Jan-06	1510	1278	232
Feb-06	2218	2105	113
Mar-06	1988	1761	227

[a]Number of non-distinct FTRs
[b]Number of distinct FTRs

4.5 Empirical Methodologies

4.5.1 Overview

In this study, the goal is to empirically test the FTR market performance under the risk-neutrality assumption. As shown earlier, a risk-neutral agent is willing to pay FTR up to his expected congestion revenue accumulated from the source to the sink specified by the FTR. Hence, under risk-neutrality, if the market is efficient, the unit cost of purchasing the FTR should be an unbiased estimator of the expected unit congestion revenue in the absence of interest. The problem is that the agents' expectations are not known and all the data we are able to collect are ex post realized value, not ex ante. To proceed, we make the following assumptions: (1) For every node, the LMP in each peak hour is independently and identically distributed (IID). So is the LMP in each off-peak hour. (2) Each market participant has perfect foresight. That is, he knows exactly the distributions of the peak and off-peak hour LMPs. Under these assumptions, we can use the realized congestion revenue as a proxy to the expected congestion revenue. Furthermore, since only those who are willing to pay more than the market clearing price can be cleared (i.e., being awarded some amount of FTR at some price), we can use this clearing price to approximate the agent's willingness to pay.

Now we are interested in testing if the clearing price of an FTR ($/MWh) effective during month t is equal to the expectation of the unit congestion revenue for that FTR. The null hypothesis we want to test is

$$H_0 : F_t^{m,n} = R_t^{m,n} \quad (7)$$

We test this hypothesis via the following regression specification:

$$R_t^{m,n} = \beta_0 + \beta_1 F_t^{m,n} + \varepsilon_t \quad (8)$$

In an efficient market with all risk-neutral agents, β_0 and β_1 should be close to 0 and 1, respectively.

In the following subsection, we first discuss briefly the linear regression model to estimate Equation (8); then move on to introduce the nonparametric kernel regression model to estimate Equation (9). Finally, we carry out a goodness-of-fit test to see if the simple linear relationship between expected congestion revenue of FTR and its clearing price could be refuted.

4.5.2 Linear Regression Model

For simple notation, let x denote the monthly FTR auction clearing price ($F_t^{m,n}$) and y denote its associated congestion revenue ($R_t^{m,n}$). Under the usual Gauss-Markov assumptions, we can specify the simple linear model as:

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i \quad (10)$$

Under ordinary least squares (OLS) estimation, the estimated coefficients $\hat{\beta}$ are:

$$\beta = \left(\mathrm{x}^T \mathrm{x}\right)^{-1} \mathrm{x}^T y \quad (11)$$

where $\beta = (\beta_0 \; \beta_1)^T$ and $\mathrm{x} = (1 \, x)$. Then the fitted linear function is

$$\hat{y} = \hat{m}_{\hat{\beta}}(x) = \mathrm{x}\hat{\beta} = \mathrm{x}\left(\mathrm{x}^T \mathrm{x}\right)^{-1} \mathrm{x}^T y \quad (12)$$

4.5.3 Kernel Regression Model

Under the weaker assumption of IID observations $(x_1, y_1)\ldots(x_n, y_n) \in R^2$, the general nonparametric regression model can be written as:

$$y_i = m(x) + \varepsilon_i \tag{13}$$

where $m(x) = E(Y \mid X = x)$ is the conditional mean function (regression function). This conditional mean function $m(\cdot)$ tells us how y and x are related "on average", which can be estimated using modern nonparametric technique such as kernel regression method. The most commonly used kernel smoothing estimator for estimating $m(\cdot)$ is called the Nadaraya-Watson (NW) estimator, which is given by

$$\hat{m}_h(x) = \frac{\sum_{i=1}^{n} K\left(\frac{x-x_i}{h}\right) y_i}{\sum_{i=1}^{n} K\left(\frac{x-x_i}{h}\right)} \tag{14}$$

where $K(\cdot)$ is the kernel function, which is usually a probability density function, and h is the smoothing parameter or bandwidth, which controls the amount of smoothness in the fitted density estimate[35]. The nice structural form of the NW estimator can be derived from the definition of the conditional expectation. See Sun (2006) for a detailed treatment.

4.5.4 Goodness-of-fit Test

To formally test whether the linear model is adequate enough to explain the relationship between expected congestion revenue of FTRs and its clearing price, we use the kernel-based nonparametric goodness-of-fit test. The null hypothesis is that the true underlying relationship between variable x and y can be represented by function m which is characterized by parameter β, that is,

$$H_0 : m = m_\beta \quad \text{v.s.} \quad H_1 : m \neq m_\beta$$

where $m_\beta(x)$ is some β-parameterized function of x. Let $\hat{m}_h(x)$ denote the NW estimator of *m(x)*, and let

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[35] Technically, a kernel function $K(\cdot)$ should satisfy the following four conditions: (i) $\int K(u)du = 1$ (pdf), (ii) $\int uK(u)du = 0$ (symmetry), (iii) $\int u^2K(u)du = \sigma_K^2 > 0$ (finite variance), and (iv) $K(u) \geq 0$ for all u in the domain of K (non-negativity).

$$\tilde{m}_{\hat{\beta}}(x) = \frac{\sum_{i=1}^{n} K\left(\frac{x-x_i}{h}\right) m_{\hat{\beta}}(x_i)}{\sum_{i=1}^{n} K\left(\frac{x-x_i}{h}\right)}$$

denote an NW smoothing of $m_{\hat{\beta}}(x)$. Note that $m_{\hat{\beta}}(x)$ is just the parametric estimate of the function $m_\beta(x)$. Hardle and Mammen (1993) propose the following test statistic:

$$T_n = nh^{1/2} \int_{-\infty}^{\infty} \{\hat{m}_h(x) - \tilde{m}_{\hat{\beta}}(x)\}^2 w(x) dx \tag{15}$$

For simplicity, let $w(x) = f(x)$ and $d = 1$ in this study, and then we can approximate T_n by:

$$T_n \approx nh^{1/2} \sum_{i=1}^{n} \{\hat{m}_h(x) - \tilde{m}_{\hat{\beta}}(x)\}^2 \tag{16}$$

In addition to calculate the test statistic, we need to find its critical value to carry out the test. Since the distribution of this test statistic does not fall into any easily-identifiable parametric distributions, there are basically two approaches to obtain its critical value -- either through asymptotic normality approximation or through bootstrap simulation. Since in this case the asymptotic approximation yields a rather inefficient speed of convergence (at the rate of $n^{-1/10}$, see Hardle and Mammen (1993) for details procedures in this approach), we opt to use the bootstrap approach to obtain the critical value. Furthermore, since naive bootstrap (i.e., resampling of $\{(x_i^*, y_i^*)\}_{i=1}^n$ from $\{(x_i, y_i)\}_{i=1}^n$ fails in regression context, we may use the wild bootstrap originally introduced by Wu (1986).

Denote $\hat{t}_\alpha^*$ as the critical value, then the bootstrap assisted GOF test for $H_0 : m = m_\beta$ is rejected if $T_n = \hat{t}_\alpha^*$. Detailed procedures about this approach can be found in Sun (2006).

4.6 Results

In this section, we discuss the results of the MISO monthly FTR auctions using the data from April 2005 through March 2006. Recall that we classify the FTR data into four categories, namely ON, OD, PN and PD. The following results are reported for all these four FTR types. First, we present some summary statistics, and calculate the degree to which risk-neutral market participants predicted congestion patterns correctly. Second, we use the linear model to estimate the effectiveness of FTRs as hedges for transmission congestion. Third, we apply non-parametric kernel model to investigate the performance of the FTR market with risk-averse agents. Finally, we evaluate the assumption of risk-neutrality using goodness-of-fit test, and the assumption of risk-aversion using the kernel smoothing method.

4.6.1 Summary Statistics

For each FTR purchase in each monthly auction between April 2005 and March 2006, we determine the unit cost of purchasing that FTR and unit revenue from holding it. Based on the data, we calculate a set of summary statistics for all the four types of FTRs over the entire sample months, and report them in Table 6-9[36].

The upper half of Table 6-9 reports the total number of observations and the average price and congestion revenues as well as their standard deviations. From the summary result, it appears that both the FTR price and congestion revenue are highly volatile, which suggests that MISO FTR market is still in its immature stage. It is worth noting that the FTR price can be negative. This can be explained as follows. Let node A and node B be the source and sink nodes, on which an FTR is defined. Then the agents anticipating congestion from node A to node B (hence with positive congestion revenue) would be willing to pay a positive amount for this FTR, while those that expect congestion in the opposite direction (hence with negative congestion revenue) would be willing to pay a negative amount for this FTR, i.e., expecting to get paid for purchasing this FTR.

As we examine the bottom half of Table 6-9, this relationship between the FTR price and congestion revenue is also confirmed by the data. For example, the positive Pearson correlation coefficient for each month indicates that F and R move together to some degree. Specifically, the average correlation is medium high at 0.54 during the sample period. We also calculate and report the number of correct prediction, which is defined as the data point where the FTR price F and the congestion revenue R have the same sign. The result shows that most market participants predict the congestion directions correctly, as the proportion of

[36] We do notice that the summary results for the first month (April 2005) are considerably different from those in the later months.

the correct predictions in the total number of awarded FTRs is always greater than 50% and for some months it is nearly 80%. Furthermore, we examine the number and the percentage of the awarded FTRs for which the price paid is lower than the congestion revenue collected. We call these FTRs "winners" in the sense that market participants can make positive profit from purchasing these FTRs. For all four types of FTRs, the percentage of "winners" is relatively high (the mean value for all these winner percentage is 59%).

4.6.2 Linear Regression Estimation

We fit the linear regression in (8) for all four types of FTR data for each month. The results are summarized in Table 10-13 and also depicted in Figure 10-13. The description of the figures is as follows: the brown dotted lines are the zero-zero lines; the red dashed line is the 45 degree line; the green line is the linear regression fit; and the blue line is the nonparametric kernel fit.

As in the figures, the results show that every quadrant has some points, although some have more points than others. For each month, there are some "wild" points far from the origin, and the observations are widely spread. In spite of that, most of the points lie close to the zero-price or zero-revenue axis, meaning that the prices and congestion revenues of most FTRs are not extremely positive or extremely negative. This is consistent with our common sense that most of the time the congestion levels are moderate.

There is clear evidence that the slope of the regression line is different from one, but is always positive, which confirms the positive correlation between the revenue and price[37]. For some months, the slope is greater than one, while for other months, it is lower than one. This means that sometimes market participants systematically lose money and sometimes systematically earn money when they try to hedge congestion risk exposures. For most of the months, the intercept is far away from zero.

These graphic observations can also be confirmed by the estimated results reported in Table 10-13. Clearly the estimated regression coefficients β_0 and β_1 are very different from 0 and 1, and most of their associated p-values are far less than 0.01. Therefore we can reject

[37] There are some cases where the slope is very close to one such as ON FTRs in Oct-05 and Mar-06.

the null hypothesis that the MISO FTR market is efficient under the assumption of risk-neutrality of the market participants.

Although the results from the linear regression model suggest that the MISO FTR market is not efficient under the assumption of risk-neutrality, it can still be true that the market is efficient if the market participants are not risk-neutral. We have shown earlier through simulation that for risk-averse agents, the efficient relationship between the FTR prices and congestion revenues would not be a 45 degree line but rather a concave function. In that case, our regression results might well be the signs of the agents being risk averse. To take other possible risk preference into account, we apply a nonparametric method, kernel regression, to estimate the relationship between FTR price and the congestion revenue.

4.6.3 Kernel Regression and GOF Test

As briefly introduced in Section 4.5.3, we applied NW kernel regression to all four types of FTR data over 12 months. The fitted kernel line (in blue) appears to be highly non-linear as they shown in Figure 10-13. Although, due to the noise data, the kernel fit does not suggest any plausible non-linear relationship between the FTR price and the congestion revenue, we still can use it to construct a goodness-of-fit (GOF) test against the linear model. As detailed in Appendix, we choose the bootstrap sample size to be 1000. The GOF test result is reported in Table 14. The result shows that all the tests are rejected at significance level 0.004 or better, which implies that the underlying relationship between F and R is significantly different from the linear fit.

The linear regression results in the preceding subsection indicate that under the assumption of risk-neutrality, the MISO FTR market does not perform efficiently, or alternatively the assumption itself is doubtable. The goodness of fit test conducted confirms us further that the linear fit is not proper for the data observed. Naturally, we are motivated to ask if the market might be efficient in the case of other risk preferences, such as risk-aversion. As shown earlier, the only thing we know about the relationship between agent's willingness to pay (F) and the expected congestion revenue from holding the FTR (ER) under risk-aversion assumption is that F>ER. From the estimated kernel regression functions, we observe that many fitted kernel curves are above the 45 degree line, which means that the risk "premium" is negative. Therefore, using the kernel fits as exploratory methods for examining the validity of the risk-aversion assumption, we may conclude that the market participants are not all risk-averse. Equivalently, the MISO FTR market is not efficient assuming that risk-aversion.

4.7 Conclusions

Some empirical work has been conducted with the existing FTR markets, such as the NYISO TCC market, but none has been done to empirically investigate the newly born MISO FTR market. In this paper, we examine the performance of this market, using publicly available data on the prices paid and congestion revenues collected by the market participants in the monthly auctions between April 2005 and March 2006. Our study provides some empirical evidence of how this young market has been performing so far. We find that the new market has something in common with the mature ones and also possess some unique features. The following lists the features that the MISO FTR market and NYISO TCC market share:

1. The correlation between FTR clearing prices and congestion revenues is positive for each month.
2. For each month, most of the FTR holders make correct predictions about the direction of congestions, that is, the price and revenue have the same sign.
3. A considerable portion of FTR holders make money by purchasing the FTRs, that is, the clearing price is lower than the revenue collected.

At the same time, the MISO FTR market has some stylized facts that are not seen in the more mature markets:

1. The number of distinct and non-distinct awards is different for different months, but increases over the 12 months on the whole.
2. The FTR market is quite thin, in the sense that there is only one buyer for most FTRs; But the difference between the number of non-distinct FTR awards and the number of distinct ones increases, meaning that this market gets thicker over time.
3. The average FTR auction clearing prices and revenues are very volatile across all months.

4. The correlation between FTR clearing prices and congestion revenues has a slightly increasing trend over the 12 months.
5. The results for the first month (April 2005) are very different from later months.

Compared with the first three common features, these five characteristics are indicative of this new market, but they could also imply that this market is getting more mature as time goes on. If we had a longer sample period, we might be able to see more clearly the progressing of this market and differentiate between the trend and seasonality. This is a task to be accomplished in the future.

As to the performance of the MISO FTR market, we find that the market works well in terms of some measures such as the proportion of correct predictions. The results from the simple linear regression seem to tell us that this market is not efficient under the assumption of risk-neutral market participants. These results, however, might indicate that the risk-neutrality assumption is not valid in the first place. Therefore, we also consider risk-averse preferences and applied nonparametric kernel fit to the data. The kernel fit results suggest that the MISO FTR market is not efficient under risk-averse assumption either. Moreover we carried out a goodness-of-fit test against the linear fit. The test results indicate that comparing with the kernel fit, the linear fit is not adequate enough to capture the underlying structure of this market.

Due to lack of more detailed data, such as the bids and offers submitted by the market participants, we cannot conclude what causes the potential inefficiency of the market. This can be accomplished in the future when more data become available.

The contributions of this paper are in two-folds. First, this paper explores the newly formed MISO FTR market using empirical methods. With the available data, we summarize the stylized facts about MISO FTR market and perform market efficiency tests. Second, in this paper we conduct theoretic analysis of the hedging role of FTRs and provide reference for further empirical analysis once we have access to more data. For example, we point out the need to know if a transaction is bilateral or via a pool when examining the risk coverage provided by FTRs. Since the data sources we now have do not provide such information, we do not differentiate between the two types in our study. We also mention that one should consider the FTR purchase in the context of the power transactions in the wholesale electricity market, but we are not able to do this in the current study due to data availability. To fully understand the underlying data generating process for the MISO FTR market, both FTRs and power transactions need to be considered.

There can be several extensions to our work. One is to compare the MISO FTR market with the more mature markets such as New England and New York FTR markets to see exactly what the difference is and why. With data of a longer sample period, we can carry out time series analysis and further investigate the development of the young market and see if it will become one similar to those long existing ones. Another extension might be to analyze FTR auctions together with the electricity transactions and individual behaviors, so as to get an integrated view of how FTRs are used to reduce the risk associated with the agent's profit. This is quite challenging and requires data not only from the FTR and energy market, but also from individual market participants.

4.8 References

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4.9 Appendices

Table 5: MISO monthly FTR allocation and auction timeline: August 2005[a]

Start Date	Stop Date	Activities[b]
	Aug. 12	MISO –Post auction model
	Aug. 15	MP[c] – Credit deadline for Sept. auction
Aug. 15	Aug. 16	MP – Allocation nominations may be submitted for Sept. monthly FTR allocation
	Aug. 19	MISO – Post Sept. monthly FTR allocation results
Aug. 22	Aug. 23	MP – Bid/offers may be submitted to buy and sell FTRs for Sept. monthly FTR auction
	Aug. 26	MISO – Post Sept. monthly FTR auction results

[a]We only list the timeline for August since all other months are very similar to August except the exact start and stop dates are slightly different across each month.
[b]Source: MISO public website http://www.midwestiso.org
[c]MP: Market participants

Table 6: Summary statistics for off-peak and non-distinct (ON) FTRs (Apr05-Mar06)

Month	Num Obs	Avg Price	Std Price	Avg Rev	Std Rev
Apr-06	88	-903.05	6310.04	53.04	172.15
May-06	174	-361.41	644.51	-32.57	1193.91
Jun-06	140	173.88	646.18	1080.48	1773.36
Jul-06	379	332.05	854.44	237.68	1110.77
Aug-06	547	134.15	748.91	1521.22	3187.88
Sep-06	336	235.65	868.86	2086.30	3328.20
Oct-06	461	26.08	467.24	467.96	1434.43
Nov-06	637	242.57	1518.96	769.04	2029.69
Dec-06	622	487.53	950.72	2030.69	3286.07
Jan-06	607	451.59	1233.12	256.18	853.17
Feb-06	857	463.42	1148.30	396.64	724.79
Mar-06	855	-14.18	948.73	-48.93	1411.83

Month	Correlation	CorrectPred	%CorrectPred	Winners	%Winners
Apr-06	0.30	48	0.55	35	0.40
May-06	0.79	151	0.87	137	0.79
Jun-06	0.67	99	0.71	93	0.66
Jul-06	0.54	250	0.66	164	0.43
Aug-06	0.64	318	0.58	418	0.76
Sep-06	0.87	230	0.68	276	0.82
Oct-06	0.31	271	0.59	310	0.67
Nov-06	0.25	395	0.62	413	0.65
Dec-06	0.44	477	0.77	463	0.74
Jan-06	0.80	489	0.81	224	0.37
Feb-06	0.59	649	0.76	408	0.48
Mar-06	0.74	639	0.75	390	0.46

Table 7: Summary statistics for off-peak and distinct (OD) FTRs (Apr05-Mar06)

Month	Num Obs	Avg Price	Std Price	Avg Rev	Std Rev
Apr-06	86	-929.43	6381.41	54.34	173.57
May-06	121	-103.08	607.90	301.77	1295.94
Jun-06	139	173.84	648.52	1078.53	1779.62
Jul-06	365	342.02	868.58	213.56	1108.39
Aug-06	502	136.59	768.43	1513.29	3171.41
Sep-06	310	251.33	902.16	2115.19	3438.81
Oct-06	428	20.88	482.21	451.04	1421.70
Nov-06	587	268.47	1575.06	799.56	2055.52
Dec-06	523	480.56	948.87	1799.63	2936.88
Jan-06	505	413.39	1302.46	219.86	897.12
Feb-06	746	515.70	1198.22	407.07	718.46
Mar-06	767	-48.20	972.58	-75.59	1470.36

Month	Correlation	CorrectPred	%CorrectPred	Winners	%Winners
Apr-06	0.30	45	0.52	36	0.42
May-06	0.75	98	0.81	85	0.70
Jun-06	0.67	98	0.71	92	0.66
Jul-06	0.55	241	0.66	151	0.41
Aug-06	0.64	294	0.59	379	0.75
Sep-06	0.88	214	0.69	254	0.82
Oct-06	0.32	249	0.58	285	0.67
Nov-06	0.24	356	0.61	373	0.64
Dec-06	0.32	395	0.76	386	0.74
Jan-06	0.81	401	0.79	178	0.35
Feb-06	0.60	573	0.77	329	0.44
Mar-06	0.76	592	0.77	373	0.49

Table 8: Summary statistics for peak and non-distinct (PN) FTRs (Apr05-Mar06)

Month	Num Obs	Avg Price	Std Price	Avg Rev	Std Rev
Apr-06	79	-6461.63	15306.82	-45.95	523.53
May-06	137	-173.27	1596.70	135.89	1147.91
Jun-06	213	1.400	962.10	889.56	1579.37
Jul-06	511	79.65	673.75	-73.46	1165.19
Aug-06	515	78.50	659.53	599.39	1495.04
Sep-06	575	-200.42	1356.63	-2390.49	10241.13
Oct-06	550	78.56	804.33	1104.68	3769.84
Nov-06	932	-22.37	1814.00	585.87	3397.96
Dec-06	1085	533.19	1424.65	2173.83	4200.80
Jan-06	903	909.33	2018.14	474.55	985.61
Feb-06	1361	602.43	1111.91	253.83	1355.96
Mar-06	1133	19.79	1128.67	90.65	1367.92

Month	Correlation	CorrectPred	%CorrectPred	Winners	%Winners
Apr-06	0.08	46	0.58	53	0.67
May-06	0.84	101	0.74	82	0.60
Jun-06	0.55	130	0.61	159	0.75
Jul-06	0.39	383	0.75	242	0.47
Aug-06	0.32	344	0.67	349	0.68
Sep-06	0.86	444	0.77	285	0.50
Oct-06	0.45	369	0.67	373	0.68
Nov-06	0.37	633	0.68	569	0.61
Dec-06	0.41	766	0.71	821	0.76
Jan-06	0.65	703	0.78	312	0.35
Feb-06	0.43	892	0.66	662	0.49
Mar-06	0.61	795	0.70	610	0.54

Table 9: Summary statistics for peak and distinct (PD) FTRs (Apr05-Mar06)

Month	Num Obs	Avg Price	Std Price	Avg Rev	Std Rev
Apr-06	78	-6313.36	15348.69	-36.13	519.53
May-06	134	-179.69	1612.36	126.22	1152.99
Jun-06	210	-0.93	968.46	900.61	1587.58
Jul-06	498	78.48	679.72	-85.65	1174.92
Aug-06	487	77.51	673.89	592.34	1481.90
Sep-06	560	-210.86	1368.89	-2511.67	10345.57
Oct-06	504	77.37	836.07	1086.22	3836.27
Nov-06	861	-38.31	1881.22	567.92	3403.62
Dec-06	923	538.81	1432.51	2016.86	4121.66
Jan-06	773	960.31	2128.17	486.67	1023.50
Feb-06	1359	602.29	1112.50	251.64	1355.52
Mar-06	994	-12.86	1170.65	63.21	1410.34

Month	Correlation	CorrectPred	%CorrectPred	Winners	%Winners
Apr-06	0.06	45	0.58	52	0.67
May-06	0.84	97	0.72	79	0.59
Jun-06	0.55	128	0.61	158	0.75
Jul-06	0.39	373	0.75	232	0.47
Aug-06	0.33	327	0.67	331	0.68
Sep-06	0.87	434	0.78	274	0.49
Oct-06	0.45	335	0.66	343	0.68
Nov-06	0.37	581	0.67	530	0.62
Dec-06	0.37	679	0.74	673	0.73
Jan-06	0.65	597	0.77	268	0.35
Feb-06	0.43	890	0.65	660	0.49
Mar-06	0.63	708	0.71	541	0.54

Table 10: Linear regression results for off-peak and non-distinct (ON) FTRs (Apr05-Mar06)

Month	Intercept	p.value	Price	p.value	resid.se	d.f.	r.sq	adj.r.sq	f.stat
Apr-06	60.32	0.00	0.01	0.01	165.40	86	0.09	0.08	8.24
May-06	498.75	0.00	1.47	0.00	728.50	172	0.63	0.63	292.65
Jun-06	760.35	0.00	1.84	0.00	1319.80	138	0.45	0.45	112.95
Jul-06	5.90	0.91	0.70	0.00	938.32	377	0.29	0.29	152.71
Aug-06	1158.01	0.00	2.71	0.00	2462.17	545	0.40	0.40	370.29
Sep-06	1302.03	0.00	3.33	0.00	1650.33	334	0.75	0.75	1028.44
Oct-06	443.01	0.00	0.96	0.00	1364.45	459	0.10	0.10	49.39
Nov-06	688.22	0.00	0.33	0.00	1967.13	635	0.06	0.06	42.10
Dec-06	1286.80	0.00	1.53	0.00	2950.91	620	0.19	0.19	150.07
Jan-06	5.13	0.82	0.56	0.00	508.31	605	0.65	0.65	1102.22
Feb-06	224.92	0.00	0.37	0.00	587.08	855	0.34	0.34	449.65
Mar-06	-33.33	0.31	1.10	0.00	951.01	853	0.55	0.55	1029.13

Table 11: Linear regression results for off-peak and distinct (OD) FTRs (Apr05-Mar06)

Month	Intercept	p.value	Price	p.value	resid.se	d.f.	r.sq	adj.r.sq	f.stat
Apr-06	61.88	0.00	0.01	0.01	166.67	84	0.09	0.08	8.19
May-06	465.74	0.00	1.59	0.00	866.36	119	0.56	0.55	149.50
Jun-06	758.46	0.00	1.84	0.00	1324.41	137	0.45	0.45	112.17
Jul-06	-27.19	0.60	0.70	0.00	925.79	363	0.30	0.30	158.75
Aug-06	1152.87	0.00	2.64	0.00	2441.04	500	0.41	0.41	345.65
Sep-06	1276.36	0.00	3.34	0.00	1663.83	308	0.77	0.77	1011.94
Oct-06	431.29	0.00	0.95	0.00	1348.16	426	0.10	0.10	48.86
Nov-06	714.96	0.00	0.32	0.00	1996.40	585	0.06	0.06	36.22
Dec-06	1320.80	0.00	1.00	0.00	2783.20	521	0.10	0.10	60.24
Jan-64	-10.64	0.67	0.56	0.00	527.26	503	0.66	0.65	956.11
Feb-06	222.08	0.00	0.36	0.00	576.08	744	0.36	0.36	414.74
Mar-06	-20.39	0.56	1.15	0.00	960.54	765	0.57	0.57	1029.91

Table 12: Linear regression results for peak and non-distinct (PN) FTRs (Apr05-Mar06)

Month	Intercept	p.value	Price	p.value	resid.se	d.f.	r.sq	adj.r.sq	f.stat
Apr-06	-28.92	0.65	0.00	0.50	525.35	77	0.01	-0.01	0.46
May-06	240.35	0.00	0.60	0.00	627.62	135	0.70	0.70	319.94
Jun-06	888.31	0.00	0.90	0.00	1326.05	211	0.30	0.30	89.74
Jul-06	-127.19	0.01	0.67	0.00	1073.96	509	0.15	0.15	91.33
Aug-06	543.26	0.00	0.72	0.00	1420.11	513	0.10	0.10	56.67
Sep-06	-1084.22	0.00	6.52	0.00	5172.01	573	0.75	0.75	1677.55
Oct-06	939.23	0.00	2.11	0.00	3370.97	548	0.20	0.20	138.61
Nov-06	601.31	0.00	0.69	0.00	3160.57	930	0.14	0.13	146.11
Dec-06	1521.93	0.00	1.22	0.00	3824.44	1083	0.17	0.17	224.85
Jan-06	185.85	0.00	0.32	0.00	749.34	901	0.42	0.42	659.49
Feb-06	-60.64	0.11	0.52	0.00	1225.91	1359	0.18	0.18	304.87
Mar-06	75.95	0.02	0.74	0.00	1081.63	1131	0.38	0.37	679.57

Table 13: Linear regression results for peak and distinct (PD) FTRs (Apr05-Mar06)

Month	Intercept	p.value	Price	p.value	resid.se	d.f.	r.sq	adj.r.sq	f.stat
Apr-06	-22.48	0.73	0.00	0.58	521.87	76	0.00	-0.01	0.31
May-06	234.26	0.00	0.60	0.00	626.52	132	0.71	0.70	318.43
Jun-06	901.45	0.00	0.90	0.00	1330.22	208	0.30	0.30	89.69
Jul-06	-138.34	0.00	0.67	0.00	1083.74	496	0.15	0.15	88.16
Aug-06	536.58	0.00	0.72	0.00	1401.80	485	0.11	0.11	58.13
Sep-06	-1130.09	0.00	6.55	0.00	5161.07	558	0.75	0.75	1688.16
Oct-06	926.62	0.00	2.06	0.00	3430.12	502	0.20	0.20	127.17
Nov-06	593.33	0.00	0.66	0.00	3168.56	859	0.13	0.13	133.33
Dec-06	1444.37	0.00	1.06	0.00	3832.41	921	0.14	0.14	145.43
Jan-06	187.79	0.00	0.31	0.00	780.76	771	0.42	0.42	555.63
Feb-06	-62.51	0.10	0.52	0.00	1225.49	1357	0.18	0.18	304.47
Mar-06	72.96	0.04	0.76	0.00	1096.48	992	0.40	0.40	650.84

Table 14: The goodness-of-fit test results for all the four types of FTRs (Apr05-Mar06)

Month	p-value.ON	p-value.OD	p-value.PN	p-value.PD
Apr-06	0.000	0.000	0.000	0.000
May-06	0.000	0.000	0.001	0.000
Jun-06	0.000	0.000	0.000	0.000
Jul-06	0.004	0.004	0.000	0.000
Aug-06	0.000	0.000	0.001	0.001
Sep-06	0.000	0.000	0.000	0.000
Oct-06	0.000	0.000	0.000	0.000
Nov-06	0.000	0.000	0.000	0.000
Dec-06	0.000	0.000	0.000	0.000
Jan-06	0.000	0.000	0.000	0.001
Feb-06	0.000	0.000	0.000	0.000
Mar-06	0.000	0.000	0.000	0.000

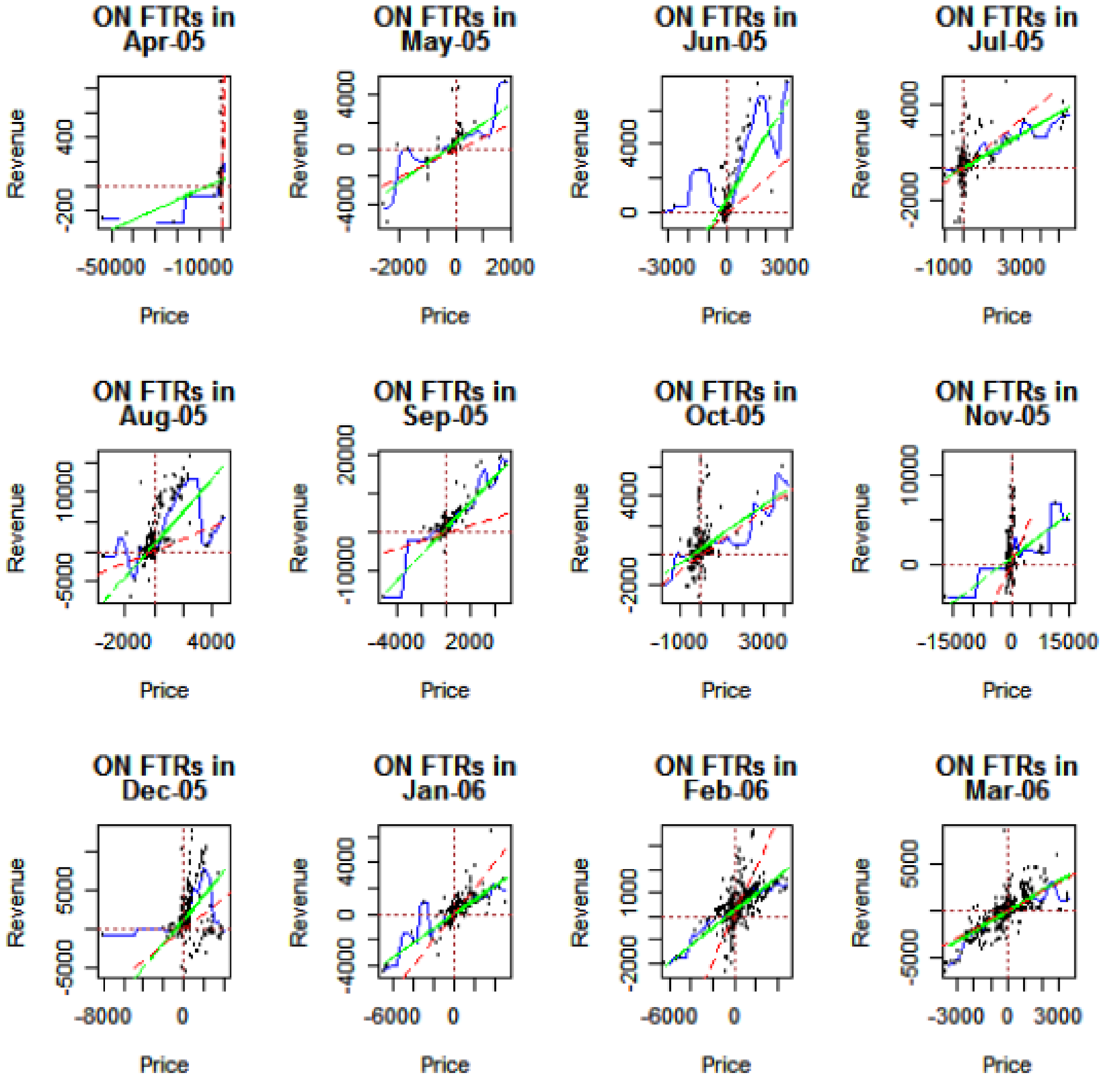

Figure 10: Linear and kernel regressions for off-peak and non-distinct (ON) FTRs

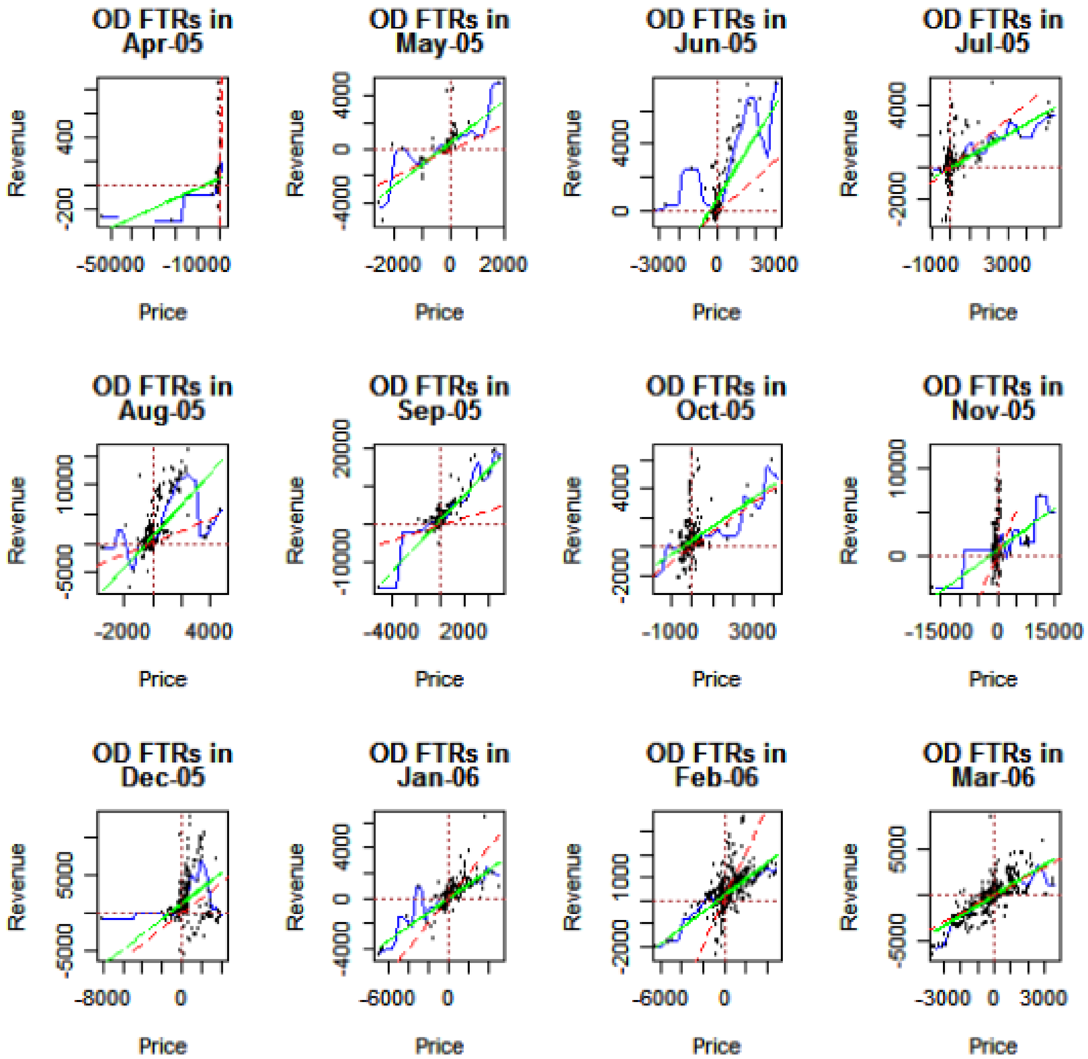

Figure 11: Linear and kernel regressions for off-peak and distinct (OD) FTRs

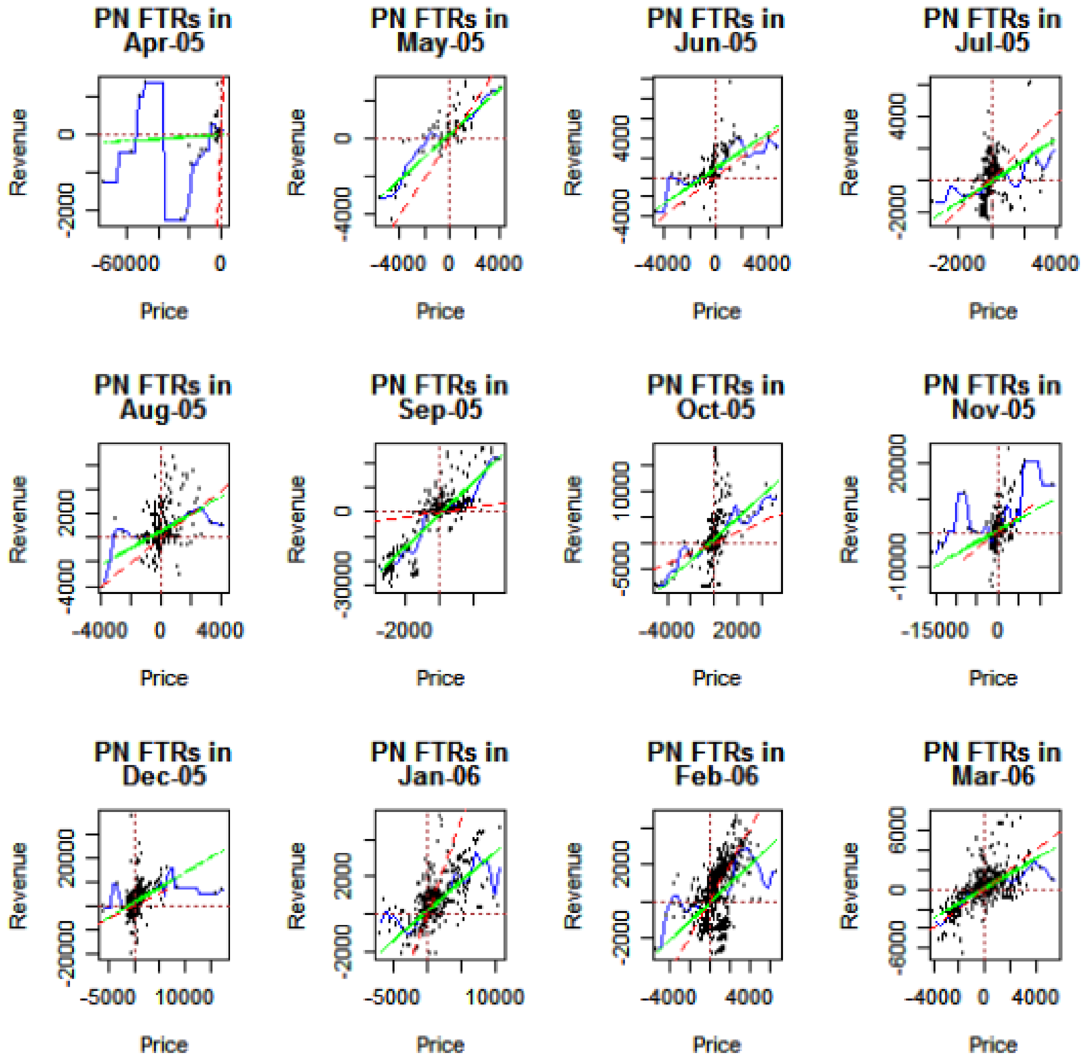

Figure 12: Linear and kernel regressions for peak and non-distinct (PN) FTRs

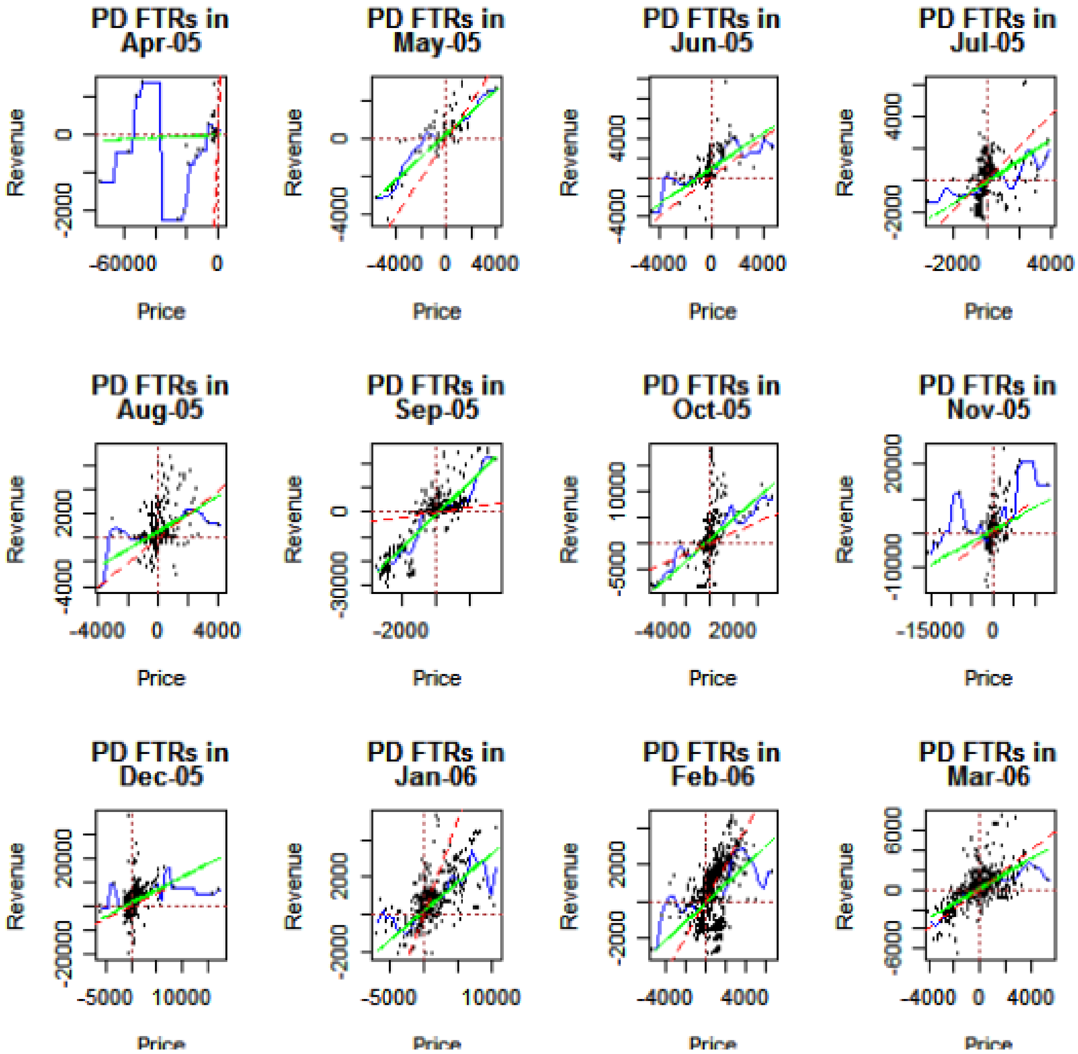

Figure 13: Linear and kernel regressions for peak and distinct (PD) FTRs

CHAPTER 5. CONCLUDING REMARKS

This dissertation covers our research findings on some issues related to the power industry including transmission investment and FTRs. Specifically, Chapters 2 and 3 investigate the efficiency attribute of transmission investment and the associated cost allocation problems, respectively. The performance of FTRs is addressed in Chapter 4.

In Chapter 2 we clarify the nature of the externality created by loop flows that can cause market-based transmission investment to be inefficient. The main conclusion is that transmission investment introduces an externality when it affects the flow of power along the lines for any given set of injections. It changes a physical aspect of power transmission, i.e. the way that electricity is transmitted in the network. As a result, the decentralized outcome may diverge from the efficient one. Specifically, the transmission investment purely induced by markets will not be optimal from society's point of view.

We say that this externality is created by loop flows, because there is no externality with transmission investment in a radial network. However, it does not mean that externality must exist whenever there are loop flows. Transmission investment that leaves the distribution factors unaffected will not cause market failure even in the presence of loop flows.

Our findings in Chapter 2 may promote understanding of transmission investment market design. The externality we investigate needs to be considered in defining property rights and developing regulation schemes in order to induce optimal investment level in transmission expansion. Mitigating market power itself is far from enough. Even if we could create perfectly competitive market environments, the outcome might still be in efficient. In Chapter 2, we suggest one way to deal with the externality, the optimal Pigouvian tax. An alternative is to establish well-defined and enforceable property rights with regard to the externality-generating activity. Then the agents themselves would reach an optimal agreement on the level of the externality through bargaining. Both options require much information, although the latter requires less than the former. In reality, it might be extremely hard to get hands on all the information needed. So an immediate extension to our work is to come up with a realistic and feasible mechanism that can remedy the externality associated with loop flows.

Since having markets make optimal transmission investment decisions is an illusion, proper regulation policies should be imposed on transmission planning. One of the related issues is who should pay for transmission investment or how the costs should be allocated, especially in the case that the investment will benefit society as a whole but hurt some agents at the same time. Intuitively, those who will gain from the investment should fund the project, but that is not all. To ensure that the project will be launched, the potential sufferers need to be compensated for their losses. In addition, we need to figure out the concrete amounts of payments, or the specific allocation of the transmission investment costs.

In Chapter 3, we address the problem above, using cooperative game theory. The main idea is that each transmission investment cost allocation problem is associated with a cooperative game. Then we can apply the solution concepts of cooperative games to the formulated game and obtain the allocation rule for the original problem. Three allocation rules are defined in Chapter 3, based on the Shapley value, core and nucleolus, respectively. Each of them provides a reasonable allocation of transmission investment cost and a benchmark of what the proper allocations of an electricity problem should be in theory. The allocation methods that are being practiced can be compared with the ideal allocations to see how far away they are from each other and how the mechanism in practice can be improved. The allocation rules we define can be used to find allocations, as long as we are able to compute the gain or loss to each agent brought about by the transmission investment project. This is not an easy task, though. Most of the time we can only estimate the expected benefits or losses from the investment, because the future use of transmission, load levels and patterns are highly uncertain. In that case, the cost allocation will be conducted based on those expected figures, instead of the deterministic ones as in the example in Chapter 3.

In Chapter 4, we conduct both theoretic and empirical studies of FTRs, a topic highly related to transmission investment. One of the major goals of this chapter is to make improvement over previous empirical work on FTR markets. A common drawback in the handful few papers in this area is the weak theoretic framework. Firstly, the authors simplify their analysis without even mentioning the underlying assumptions, but their methodologies are not proper if those assumptions do not hold. The empirical results are interpreted as if those assumptions were valid, which makes their conclusions less convincing. Secondly, the methodology in some of the papers is based on some unproved conjectures. For example, one of the papers asserts that for risk-averse agents, the congestion revenue from an FTR is a concave function of the price paid for it. Whether this is true has never been shown. Thirdly, the general risk hedging theory is directly borrowed for the FTR analysis without considering the specific features of the power market. For example, electricity transactions can be made via a power pool or according to a bilateral contract, and the hedging of FTR has different manifestations in these two types of transactions. None of the existing papers takes this into account. In comparison, our paper, in the first place, constructs a more complete and tenable theoretic framework than those in the literature. The unproved conjecture is also demonstrated in the paper. Besides, we emphasize the importance of understanding the data generation process in addition to working with the observations and suggest what other data need to be used for a more complete analysis.

We come up with stylized facts about this young market: the number of FTR awards is increasing overtime, the market is rather thin, but has sign of getting thicker, and the FTR clearing prices and revenues are highly volatile etc. These features are not observed in the more mature market like the NYISO FTR market, and have not been addressed before. We also find that the MISO FTR market is not efficient under both risk-neutral in terms of the relationship between the payment and revenue.

This dissertation addresses some open questions that the U.S. power industry faces and elucidates our ideas about them. This does not end the debate on the issues and further investigations may be well needed. Besides, many other issues should also be explored in the future. We hope to proceed with efforts in the research of the power industry and make our contributions to its development.